

Maximal abelian star subalgebra

`MathlibAnnex.Analysis.CStarAlgebra.IsMaximalAbelian`

Defines maximal abelian unital star subalgebras by commutativity and maximality under inclusion.

Statement

Let D be a unital star subalgebra of a unital complex C^* -algebra A . D is commutative, and any commutative unital star subalgebra E containing D is contained in D ; thus D is maximal for inclusion among such subalgebras.

Definition

$\text{IsMaximalAbelian } D \leftrightarrow \text{IsMulCommutative } D \wedge \forall E, \text{IsMulCommutative } E \rightarrow D \leq E \rightarrow E \leq D.$

Assumptions

Let D be a unital star subalgebra of a unital complex C^* -algebra A .

Conclusion

D is commutative, and any commutative unital star subalgebra E containing D is contained in D ; thus D is maximal for inclusion among such subalgebras.

Main citations

Lean source signature (exact)

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def IsMaximalAbelian (D : StarSubalgebra ℂ A) : Prop
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[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: 005d767a3193a30c065ad807166f2e150c8ac3bbd28de67fd113b74463f676eb

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