

No nonzero irreducible closed subrepresentation of the tracial model

`MathlibAnnex.CStarAlgebra.CAR.not_isIrreducible_restrictToReducing_separableCounterexampleRepresentation`

Extends the separable irreducibility exclusion to every closed reducing subspace of the fixed representation.

Statement

The faithful tracial representation $\rho: A \rightarrow B(H\tau)$ has no nonzero irreducible closed subrepresentation.

Assumptions

Let $A \subseteq B(H_{at})$ be the fixed unital C^* -algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C , and then taking the norm-closed unital $*$ -algebra they generate. Let $H\tau = L^2(C, \tau C)$ be the GNS Hilbert space of the normalized CAR trace τC . Let ρ be the specified tracial representation, and let M be a closed complex-linear subspace of $H\tau$ invariant under every $\rho(a)$ and its adjoint.

Conclusion

The restricted representation $\rho|_M: A \rightarrow B(M)$ is not both nonzero and irreducible. This includes $M = \{0\}$, for which nonzeroness already fails.

Proof route

A closed subspace M is complete, and as a subspace of the separable Hilbert space $H\tau$ it is separable. Apply the exclusion of separable irreducible representations to the exact restriction.

Proof steps

1. Use closedness of M to equip it with its complete inherited Hilbert-space structure.
2. Use separability of $H\tau$ to obtain separability of M .
3. Apply the universal no-separable-irreducible-representation theorem to $\rho|_M$.

Lean realization notes

The source argument $hM : p.Reduces\ M$ contains closedness and invariance under both operators and their adjoints. The conclusion is about the defined restriction, not an arbitrary subset or a nonclosed algebraic subspace. It does not assert that all invariant subspaces are absent.

Language: en · CARD_CONTENT_COMPLETE

Main citations

[MathlibAnnex.CStarAlgebra.CAR.not_isIrreducible_of_separable](#)

Lean source declaration (exact)

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theorem not_isIrreducible_restrictToReducing_separableCounterexampleRepresentation
(M : Submodule ℂ SeparableCounterexampleHilbertSpace)
(hM : separableCounterexampleRepresentation.Reduces M) :
letI : CompleteSpace M := hM.1.completeSpace_coe
¬ Representation.IsIrreducible
(Representation.restrictToReducing separableCounterexampleRepresentation M hM) := by
letI : CompleteSpace M := hM.1.completeSpace_coe
letI : TopologicalSpace.SeparableSpace SeparableCounterexampleHilbertSpace :=
separableSpace_separableCounterexampleHilbertSpace
letI : TopologicalSpace.SeparableSpace M := inferInstance
exact not_isIrreducible_of_separable
(Representation.restrictToReducing
separableCounterexampleRepresentation M hM).toNonUnitalStarAlgHom
```

[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: 0a759cbd58d457ef91977667285e6fa6787a5451b1961b92e7c30aac6d0d0d04

Card SHA-256: d76eb8815b2cd36bec71c53984e9ed201889ea48c67af6aa2ea2a3b7f1ddcd8b

Approved exposition SHA-256: 16a2e7290f22110200ea5359936372c57675e7483af18e0af60d7d74d349d093

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