

# Representative of the unique irreducible-representation class

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.IsSingletonIrreducibleModel`

Defines when a nonzero irreducible representation  $\pi$  is a representative of the unique unitary-equivalence class of nonzero irreducible representations of  $A$ .

## Statement

Let  $A$  be a complex  $C^*$ -algebra, not assumed unital,  $H$  a complex Hilbert space, and  $\pi$  a representation of  $A$  on  $H$ . The representation  $\pi$  is a representative of the unique unitary-equivalence class of nonzero irreducible representations of  $A$ :  $\pi$  is nonzero and irreducible, and every nonzero irreducible representation of  $A$  on any complex Hilbert space is unitarily equivalent to  $\pi$ . The definition does not assume faithfulness or separability of  $A$  or  $H$ .

## Definition

$\text{IsSingletonIrreducibleModel } \pi \leftrightarrow \text{IsIrreducible } \pi \wedge \forall K \rho, \text{IsIrreducible } \rho \rightarrow \text{UnitaryEquivalent } \pi \rho.$

## Assumptions

Let  $A$  be a complex  $C^*$ -algebra, not assumed unital,  $H$  a complex Hilbert space, and  $\pi$  a representation of  $A$  on  $H$ .

## Conclusion

The representation  $\pi$  is a representative of the unique unitary-equivalence class of nonzero irreducible representations of  $A$ :  $\pi$  is nonzero and irreducible, and every nonzero irreducible representation of  $A$  on any complex Hilbert space is unitarily equivalent to  $\pi$ . The definition does not assume faithfulness or separability of  $A$  or  $H$ .

## Main citations

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.IsIrreducible`

Exact formal dependency; inspect the linked Card and exact source.

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.UnitaryEquivalent`

Exact formal dependency; inspect the linked Card and exact source.

## Lean source signature (exact)

```
def IsSingletonIrreducibleModel
(pi : NonUnitalCStarRepresentation A H) : Prop
```

Read exact [MathlibAnnex v0.4.0 source](#)

## Exact Card identity

Stable Card ID: 15b02dd39a6c9afbc2cc1f023c8e4b29cc806759b53b21c82065a410d00117ff

Card SHA-256: c089732cc79ef2a1a463e42c0bd496ccda8b2525b493dc8bcde5e4da70950bdb

Approved exposition SHA-256: 692de46deb53242bcac4b1e3ae634370f422245d1a38f6efdf00ff80fa583aec

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source-exposition correspondence, not an independent proof audit.