

Pure state as an extreme point

`MathlibAnnex.Analysis.CStarAlgebra.IsPureState`

Defines pure states as the real extreme points of the state space.

Statement

Let A be a unital complex C^* -algebra and let $\varphi : A \rightarrow \mathbb{C}$ be a continuous complex-linear functional. φ is pure exactly when it belongs to the real extreme points of the normalized positive state space `stateSpace A`. Membership in that extreme-point set includes membership in the state space.

Definition

`IsPureState A  $\varphi \leftrightarrow \varphi \in (\text{stateSpace } A).\text{extremePoints } \mathbb{R}$` .

Assumptions

Let A be a unital complex C^* -algebra and let $\varphi : A \rightarrow \mathbb{C}$ be a continuous complex-linear functional.

Conclusion

φ is pure exactly when it belongs to the real extreme points of the normalized positive state space `stateSpace A`. Membership in that extreme-point set includes membership in the state space.

Main citations

Lean source signature (exact)

```
def IsPureState (A : Type u) [CStarAlgebra A] [PartialOrder A] [StarOrderedRing A]
  (phi : A →L[ $\mathbb{C}$ ]  $\mathbb{C}$ ) : Prop
```

[Read exact MathlibAnnex v0.4.0 source](#)

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