

Unbundled compact-operator model

`MathlibAnnex.Analysis.CStarAlgebra.IsCompactOperatorModel`

Packages injectivity, compact-valuedness, and surjectivity onto all compact operators into one unbundled predicate.

Statement

Let A be a complex C^* -algebra, not assumed unital, H a complex Hilbert space, and e a nonunital complex star-algebra homomorphism from A to the bounded operators on H . The model predicate consists of all three clauses: e is injective; every represented $e(a)$ is compact; and each compact bounded operator T on H equals $e(a)$ for some a in A .

Definition

$\text{IsCompactOperatorModel } e \leftrightarrow \text{Injective } e \wedge (\forall a, \text{IsCompactOperator } (e\ a)) \wedge \forall T, \text{IsCompactOperator } T \rightarrow \exists a, e\ a = T.$

Assumptions

Let A be a complex C^* -algebra, not assumed unital, H a complex Hilbert space, and e a nonunital complex star-algebra homomorphism from A to the bounded operators on H .

Conclusion

The model predicate consists of all three clauses: e is injective; every represented $e(a)$ is compact; and each compact bounded operator T on H equals $e(a)$ for some a in A .

Main citations

Lean source signature (exact)

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def IsCompactOperatorModel [NonUnitalCStarAlgebra A]
  (e : A →*na [C] (H →L[C] H)) : Prop
```

Read exact [MathlibAnnex v0.4.0 source](#)

Exact Card identity

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