

The underlying set of A has size at most the continuum

`MathlibAnnex.CStarAlgebra.CAR.cardinalMk_atomicCounterexampleAlgebra_le_continuum`

Obtains a carrier-cardinality bound from a faithful representation on a separable Hilbert space.

Statement

The underlying set of the fixed CAR-based C^* -algebra A has cardinality at most $\mathfrak{c} = 2^{\aleph_0}$.

Assumptions

Let $A \subseteq B(H_{\text{at}})$ be the fixed unital C^* -algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C , and then taking the norm-closed unital $*$ -algebra they generate. No additional hypotheses are supplied to this theorem. Its proof uses the previously constructed faithful representation on the separable CAR trace-GNS space.

Conclusion

$|A| \leq \mathfrak{c}$.

Proof route

Apply the cardinality bound for an algebra admitting an injective representation on a separable Hilbert space.

Proof steps

1. Equip the fixed trace-GNS space with its established separable-space structure.
2. Use injectivity of the tracial representation and the operator-space cardinality bound.

Lean realization notes

Cardinal.mk here refers to the underlying set, not the density character or a vector-space dimension. No CH premise is used. The lower bound required for equality is supplied separately.

Language: en · CARD_CONTENT_COMPLETE

Main citations

MathlibAnnex.Analysis.CStarAlgebra.Representation.cardinalMk_le_continuum_of_injective

Lean source declaration (exact)

```
/-- The existing faithful separable representation bounds the fixed carrier. -/  
theorem cardinalMk_atomicCounterexampleAlgebra_le_continuum :  
#AtomicCounterexampleAlgebra ≤ Cardinal.continuum := by  
letI : TopologicalSpace.SeparableSpace SeparableCounterexampleHilbertSpace :=  
separableSpace_separableCounterexampleHilbertSpace  
exact Representation.cardinalMk_le_continuum_of_injective  
separableCounterexampleRepresentation separableCounterexampleRepresentation_injective
```

[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: 2d084b17a142cd92b0c25e448adec6b0e58bfc7280434250e46a36acbc881a4b

Card SHA-256: 02df2eeba0368fda2c5814cec462bc5566696823c4b4dd7fe316f492ef6c26d6

Approved exposition SHA-256: 2819956fe322be85a3bf072b0a1b2f513523a360b7b479174c28a5e0114d7c2d

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