The dependent co-sum of compact-operator algebras

`MathlibAnnex.Analysis.CStarAlgebra.CompactOperatorC0Sum`

abbrev

Names the c_0-sum of a family of compact-operator algebras.

Statement

Let $(H_i)_{i\in\iota}$ be any family of complete complex Hilbert spaces. This abbreviation denotes the c_0-sum of the compact-operator algebras $\mathcal{K}(H_i)$.

Definition

Use the dependent c_0-sum with fiber $A_i = \mathcal{K}(H_i)$, the compact operators on the given H_i. Its elements are bounded families $T = (T_i)$, $T_i \in \mathcal{K}(H_i)$, such that for every $\varepsilon > 0$ only finitely many i have $\varepsilon \le \|T_i\|$. Its norm is $\sup_i \|T_i\|$ and its algebra operations are coordinatewise. The RHS specifies precisely this family of subalgebras of bounded operators.

Assumptions

Every H_i has its own normed additive group, complex inner product and completeness. Zero and infinite-dimensional Hilbert spaces are allowed. There is no restriction on the size of the index set.

Conclusion

The output is an abbreviation for this dependent algebra. It does not denote all compact operators on a Hilbert-space sum and does not include additional off-diagonal operator entries.

Main citations

- Exact complete definition.
- C0DirectSum: the defining abbreviation.

Lean source signature (exact)

The following is the complete defining right-hand side, copied from the fixed source.

```
abbrev CompactOperatorC0Sum {ι : Type uι} (H : ι → Type uH)
    [∀ i, NormedAddCommGroup (H i)] [∀ i, InnerProductSpace ℂ (H i)]
    [∀ i, CompleteSpace (H i)] :=
  C0DirectSum
    (fun i => ContinuousLinearMap.compactOperatorStarSubalgebra ℂ (H i))
```

In the source	Mathematical meaning
Exact declaration `H : ι → Type uH`	The family of Hilbert spaces H_i; the spaces may vary with i.
The three **Exact declaration** `∀ i` instances	Each H_i has the stated norm and complex inner product and is complete.
Exact declaration `ContinuousLinearMap.compactOperatorStarSubalgebra ℂ (H i)`	The subalgebra $\mathcal{K}(H_i)$ of compact complex-linear endomorphisms of this same fiber.
Exact declaration `C0DirectSum (fun i => ...)`	The preceding dependent vanishing-at-infinity construction applied to exactly those algebras.

Exact surrounding source context (separate excerpts)

Source lines 23–25:

```
namespace MathlibAnnex.Analysis.CStarAlgebra

universe uι uA uH
```