

Scalar character of the minimal unitization

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.infinityCharacter`

Defines the distinguished scalar-coordinate character of the minimal unitization.

Statement

Let A be a complex C^* -algebra, not assumed unital, and form its minimal unitization $\text{Unitization } \mathbb{C} A$. The infinity character is the character induced by projection onto the scalar coordinate: it evaluates a unitization element at its $\mathbb{C}$ component. It is a character of $\text{Unitization } \mathbb{C} A$, not of A itself.

Definition

`infinityCharacter = CharacterSpace.equivAlgHom.symm (Unitization.fstHom  $\mathbb{C} A$ );`
`infinityCharacter(c,a)=c.`

Assumptions

Let A be a complex C^* -algebra, not assumed unital, and form its minimal unitization $\text{Unitization } \mathbb{C} A$.

Conclusion

The infinity character is the character induced by projection onto the scalar coordinate: it evaluates a unitization element at its $\mathbb{C}$ component. It is a character of $\text{Unitization } \mathbb{C} A$, not of A itself.

Main citations

Lean source signature (exact)

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noncomputable def infinityCharacter :  
WeakDual.characterSpace  $\mathbb{C}$  (Unitization  $\mathbb{C} A$ )
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[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

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AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.