MATHLIBANNEX / CANONICAL DECLARATION CARD

# Eigenvector for a character distinct from the scalar character

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists_unit_eigenvector_of_character_ne_infinity`

Produces a unit joint eigenvector whose eigenvalue character is the prescribed character distinct from the scalar character.

## Statement

Let A be a nonzero complex C*-algebra, not assumed unital; let H be a complex Hilbert space; let π be a representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A; let D be a closed unital star subalgebra of Unitization ℂ A; and let χ be a character of D distinct from infinityCharacterOn D. No separability assumption on H is required. There is a unit vector η in H such that π.unitization(d)η=χ(d)η for every d in D.

## Assumptions

Let A be a nonzero complex C*-algebra, not assumed unital; let H be a complex Hilbert space; let π be a representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A; let D be a closed unital star subalgebra of Unitization ℂ A; and let χ be a character of D distinct from infinityCharacterOn D. No separability assumption on H is required.

## Conclusion

There is a unit vector η in H such that π.unitization(d)η=χ(d)η for every d in D.

## Proof idea

The source extends χ to a pure state on the unitization, forms its GNS representation, and shows its restriction to A is nonzero because χ is distinct from the scalar character. The unique-class hypothesis gives unitary equivalence to π and transfers the normalized GNS cyclic vector to η, preserving the joint eigenvector equation.

## Proof steps

1. Exact Lean statement: ∀ {A : Type u} [inst : NonUnitalCStarAlgebra A] [inst_1 : PartialOrder A] [StarOrderedRing A] {H : Type v} [inst_3 : NormedAddCommGroup H] [inst_4 : InnerProductSpace ℂ H] [inst_5 : CompleteSpace H] [Nontrivial A] (pi : MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation A H), pi.IsSingletonIrreducibleModel → ∀ (D : StarSubalgebra ℂ (Unitization ℂ A)) [inst_7 : IsClosed ↑D] (chi : ↑(WeakDual.characterSpace ℂ ↥D)), chi ≠ MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.infinityCharacterOn D → ∃ eta, ‖eta‖ = 1 ∧ ∀ (d : ↥D), (pi.unitization ↑d) eta = chi d • eta

2. ∃ η : H, ‖η‖=1 ∧ ∀ d : D, π.unitization d η = χ d • η.

3. The source extends χ to a pure state on the unitization, forms its GNS representation, and shows its restriction to A is nonzero because χ is distinct from the scalar character. The unique-class hypothesis gives unitary equivalence to π and transfers the normalized GNS cyclic vector to η, preserving the joint eigenvector equation.

## Main citations

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.IsSingletonIrreducibleModel

Exact formal dependency; inspect the linked Card and exact source.

MathlibAnnex.Analysis.CStarAlgebra.exists_pureState_extension

Exact formal dependency; inspect the linked Card and exact source.

MathlibAnnex.Analysis.CStarAlgebra.isIrreducible_pureState_gnsStarAlgHom

Exact formal dependency; inspect the linked Card and exact source.

## Lean source signature (exact)

```
theorem exists_unit_eigenvector_of_character_ne_infinity [Nontrivial A]
(pi : NonUnitalCStarRepresentation A H)
(hsingle : IsSingletonIrreducibleModel.{u, v, u} pi)
(D : StarSubalgebra ℂ (Unitization ℂ A))
[IsClosed (D : Set (Unitization ℂ A))]
(chi : WeakDual.characterSpace ℂ D)
(hchi : chi ≠ infinityCharacterOn (A := A) D) :
∃ eta : H, ‖eta‖ = 1 ∧
∀ d : D, pi.unitization (d : Unitization ℂ A) eta = chi d • eta
```

Read exact MathlibAnnex v0.4.0 source

## Exact Card identity

Stable Card ID: 7a1b136b8464e6029397491d4d05fa22de45ecb383b3384c4e0c2fcf7df3150a

Card SHA-256: 66362986cb8e27b76374628c9b58a049b33fc3ce262e58e3e38d04a93a47ed08

Approved exposition SHA-256: 2fa0f1907d615d98cb0abc7d5f710976c87fe7d6a46918c4932bba8991d17f25

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.