Compact-operator model from a unital representative of the unique irreducible-representation class

`MathlibAnnex.Analysis.CStarAlgebra.Representation.faithful_and_compactOperatorModel_of_singleton_amongNonUnital`

Shows that a separably acting unital representative of the unique irreducible-representation class is faithful and has image exactly K(H).

Statement

Let A be a nonzero unital complex C*-algebra, H a separable complex Hilbert space, and π a unital representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A. π is injective; its underlying nonunital star homomorphism is injective, maps every a∈A to a compact operator, and represents every compact operator on H.

Assumptions

Let A be a nonzero unital complex C*-algebra, H a separable complex Hilbert space, and π a unital representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A.

Conclusion

π is injective; its underlying nonunital star homomorphism is injective, maps every a∈A to a compact operator, and represents every compact operator on H.

Proof idea

The source passes from the nonunital-interface formulation of the unique-class condition to its unital formulation and applies the unital faithful-and-compact-model theorem. The model entails injectivity, compactness of each image and a preimage for every compact T; no bundled StarAlgEquiv is produced.

Proof steps

1. Exact Lean statement: ∀ {A : Type u} [inst : CStarAlgebra A] [inst_1 : PartialOrder A] [StarOrderedRing A] {H : Type v} [inst_3 : NormedAddCommGroup H] [inst_4 : InnerProductSpace ℂ H] [inst_5 : CompleteSpace H] [Nontrivial A] [TopologicalSpace.SeparableSpace H] (pi : MathlibAnnex.Analysis.CStarAlgebra.Representation A H), pi.IsSingletonIrreducibleModelAmongNonUnital → Function.Injective ⇑pi ∧ MathlibAnnex.Analysis.CStarAlgebra.IsCompactOperatorModel (StarAlgHom.toNonUnitalStarAlgHom pi)

2. Injective π ∧ IsCompactOperatorModel π.toNonUnitalStarAlgHom.

3. The source passes from the nonunital-interface formulation of the unique-class condition to its unital formulation and applies the unital faithful-and-compact-model theorem. The model entails injectivity, compactness of each image and a preimage for every compact T; no bundled StarAlgEquiv is produced.

Main citations

MathlibAnnex.Analysis.CStarAlgebra.IsCompactOperatorModel

Exact formal dependency; inspect the linked Card and exact source.

MathlibAnnex.Analysis.CStarAlgebra.Representation.IsSingletonIrreducibleModelAmongNonUnital

Exact formal dependency; inspect the linked Card and exact source.

MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional_space_of_singleton

Exact formal dependency; inspect the linked Card and exact source.

MathlibAnnex.Analysis.CStarAlgebra.Representation.surjective_of_irreducible_of_finiteDimensional

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

```
theorem faithful_and_compactOperatorModel_of_singleton_amongNonUnital
[Nontrivial A] [TopologicalSpace.SeparableSpace H]
(pi : Representation A H)
(hsingle : IsSingletonIrreducibleModelAmongNonUnital.{u, v, u} pi) :
Function.Injective pi ∧
IsCompactOperatorModel pi.toNonUnitalStarAlgHom
```

Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: 8b10daad485fe797ba4250420c41ce6ffba033b4085e1a302b3eed0c4dd5d141

Card SHA-256: 16bca310f77405e367eea080da63ce9a180e242a66c01b35672b2def0606a50d

Approved exposition SHA-256: 0ee3edfacd50b3fe2a8ed36327c49d9a5114c7900c84979abe95225482d9f110

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.