The underlying set of A has cardinality continuum

`MathlibAnnex.CStarAlgebra.CAR.cardinalMk_atomicCounterexampleAlgebra`

Identifies the exact cardinality of the same fixed algebra.

Statement

The underlying set of the fixed CAR-based C*-algebra A has cardinality exactly $\mathfrak{c} = 2^{\aleph_0}$.

Assumptions

Let $A \subseteq B(H_{at})$ be the fixed unital C*-algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C, and then taking the norm-closed unital *-algebra they generate. This is an unconditional theorem about that fixed construction, with no CH assumption.

Conclusion

$|A| = \mathfrak{c}$.

Proof route

Combine the upper bound from the faithful separable representation with the dense-set lower bound applied to the whole space A.

Proof steps

1. Use the upper bound for the cardinality of A to obtain |A| ≤ 𝔠.
2. The whole carrier is norm dense in itself, so the lower bound for all norm-dense subsets of A gives 𝔠 ≤ |A|.
3. Conclude equality by antisymmetry.

Lean realization notes

This declaration concerns carrier cardinality. The exact norm-density theorem for A is a separate result; neither is a statement about Hamel dimension.

Language: en · CARD_CONTENT_COMPLETE

Main citations

MathlibAnnex.CStarAlgebra.CAR.cardinalMk_atomicCounterexampleAlgebra_le_continuum

MathlibAnnex.CStarAlgebra.CAR.continuum_le_cardinalMk_dense_atomicCounterexampleAlgebra

Lean source declaration (exact)

```
/-- The cardinality of the underlying set of the fixed algebra is continuum. -/
theorem cardinalMk_atomicCounterexampleAlgebra :
#AtomicCounterexampleAlgebra = Cardinal.continuum := by
apply le_antisymm cardinalMk_atomicCounterexampleAlgebra_le_continuum
simpa using continuum_le_cardinalMk_dense_atomicCounterexampleAlgebra Set.univ dense_univ
```

Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: 8f6080ffedd7a05589c15beb77da0914b520154a8e6de93f7d47d064cf365ca0

Card SHA-256: 75a7668379333ad8738557c1da68955398f298726394d3d716d30ce59bcf1ad3

Approved exposition SHA-256: 03204453e9983c2017334f0751a568be13220a47231adfbbf104e8d089a2093a

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.