

# Algebraic compact-preimage ideal

`MathlibAnnex.CStarAlgebra.compactPreimageIdeal`

Collects the elements represented by compact operators into a two-sided algebraic ideal.

## Statement

Let  $A$  be a complex  $C^*$ -algebra, not assumed unital,  $H$  a complex Hilbert space, and  $\rho$  a representation of  $A$  on  $H$ . `compactPreimageIdeal  $\rho$`  is the two-sided algebraic ideal of elements whose represented bounded operator is compact. Norm-closedness is a separate theorem and is not a field of this returned `TwoSidedIdeal`.

## Definition

$(\text{compactPreimageIdeal } \rho : \text{Set } A) = \{a : A \mid \text{IsCompactOperator } (\rho \ a)\}.$

## Assumptions

Let  $A$  be a complex  $C^*$ -algebra, not assumed unital,  $H$  a complex Hilbert space, and  $\rho$  a representation of  $A$  on  $H$ .

## Conclusion

`compactPreimageIdeal  $\rho$`  is the two-sided algebraic ideal of elements whose represented bounded operator is compact. Norm-closedness is a separate theorem and is not a field of this returned `TwoSidedIdeal`.

## Main citations

## Lean source signature (exact)

```
def compactPreimageIdeal [NonUnitalCStarAlgebra A]
  (rho : A →*na [C] (H →L [C] H)) : TwoSidedIdeal A
```

Read exact [MathlibAnnex v0.4.0 source](#)

## Exact Card identity

Stable Card ID: 9355a7df6defbdd54a898d97fa5d5ff61ac59614adb1ceee96df088d9c1a66b6

Card SHA-256: ee21384769079014e7f0b83108c683e777408b5e63149385e2931aa44db2b8b1

Approved exposition SHA-256: d1c18c74316e9f18633622fc81204fd774025015ae12d9e8a5d1bb9d33a83cde

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