Scalar corner from a representative of the unique irreducible-representation class

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists_nonzero_projection_scalar_corner`

Produces a nonzero projection with one-dimensional corner from a separably acting representative of the unique irreducible-representation class.

Statement

Let A be a nonzero complex C*-algebra, not assumed unital, H a separable complex Hilbert space, and π a representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A. There is a nonzero star projection p in A whose every corner p a p is a complex scalar multiple of p.

Assumptions

Let A be a nonzero complex C*-algebra, not assumed unital, H a separable complex Hilbert space, and π a representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A.

Conclusion

There is a nonzero star projection p in A whose every corner p a p is a complex scalar multiple of p.

Proof idea

The source takes a nonzero a, inserts b=a* a into a maximal abelian subalgebra of the unitization, uses separability to count its characters, and obtains an isolated character distinct from the scalar character. A projection supported at that character has a scalar corner; the zero scalar coordinate forces that projection back into A.

Proof steps

1. Exact Lean statement: ∀ {A : Type u} [inst : NonUnitalCStarAlgebra A] [inst_1 : PartialOrder A] [StarOrderedRing A] {H : Type v} [inst_3 : NormedAddCommGroup H] [inst_4 : InnerProductSpace ℂ H] [inst_5 : CompleteSpace H] [Nontrivial A] [TopologicalSpace.SeparableSpace H] (pi : MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation A H), pi.IsSingletonIrreducibleModel → ∃ p, IsStarProjection p ∧ p ≠ 0 ∧ ∀ (a : A), ∃ c, p * a * p = c • p

2. ∃ p : A, IsStarProjection p ∧ p≠0 ∧ ∀ a : A, ∃ c : ℂ, p*a*p=c•p.

3. The source takes a nonzero a, inserts b=a* a into a maximal abelian subalgebra of the unitization, uses separability to count its characters, and obtains an isolated character distinct from the scalar character. A projection supported at that character has a scalar corner; the zero scalar coordinate forces that projection back into A.

Main citations

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.countable_characterSpace_of_nonUnital_singleton

Exact formal dependency; inspect the linked Card and exact source.

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists_isolated_character_ne_infinity

Exact formal dependency; inspect the linked Card and exact source.

MathlibAnnex.Analysis.CStarAlgebra.exists_maximalAbelian_containing_isSelfAdjoint

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

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theorem exists_nonzero_projection_scalar_corner [Nontrivial A]
[TopologicalSpace.SeparableSpace H]
(pi : NonUnitalCStarRepresentation A H)
(hsingle : IsSingletonIrreducibleModel.{u, v, u} pi) :
∃ p : A, IsStarProjection p ∧ p ≠ 0 ∧
∀ a : A, ∃ c : ℂ, p * a * p = c • p
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Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: 964e8b0b51be9a3fc80a3f61b3bcb1b8cf70461ce4137aed6128c4ced9074130

Card SHA-256: 5b07a7c7de32145f121f2a50d936d721f077b048bca799b7e2852db18b512bba

Approved exposition SHA-256: 6e9c1ad5c1903b5edc14e35ad7be7de2fbf796f0190ed8a16086f39929864c49

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.