An isolated point in a countable Baire space

`MathlibAnnex.Topology.exists_isOpen_singleton`

Extracts an isolated point from a nonempty countable T1 Baire space.

Statement

Let X be nonempty, countable, T1 and Baire. At least one singleton {x} is open in X.

Assumptions

Let X be nonempty, countable, T1 and Baire.

Conclusion

At least one singleton {x} is open in X.

Proof idea

T1 makes each singleton closed. The countable union of those closed singletons is X; Mathlib's Baire closed-cover lemma gives one with nonempty interior. Since that interior lies inside a singleton, the singleton is open.

Proof steps

1. Exact Lean statement: ∀ {X : Type u} [inst : TopologicalSpace X] [Nonempty X] [Countable X] [T1Space X] [BaireSpace X], ∃ x, IsOpen {x}

2. ∃ x : X, IsOpen ({x} : Set X).

3. T1 makes each singleton closed. The countable union of those closed singletons is X; Mathlib's Baire closed-cover lemma gives one with nonempty interior. Since that interior lies inside a singleton, the singleton is open.

Main citations

Lean source signature (exact)

```
theorem exists_isOpen_singleton [Nonempty X] [Countable X] [T1Space X]
[BaireSpace X] : ∃ x : X, IsOpen ({x} : Set X)
```

Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: 9c5012403e7138fc2010b82937c3665b67957413b75f52e1a4527ad36e819be4

Card SHA-256: 819701b501957cdcf19003e34a0cad05ad728f597d7dc5224d359463cbf86461

Approved exposition SHA-256: e7ebb402978675eecd0e30a14444040541ac38228fa9eb4be2b5a22c3f488f53

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.