

The fixed CAR-based C^* -algebra

`MathlibAnnex.CStarAlgebra.CAR.AtomicCounterexampleAlgebra`

Defines the one algebra used throughout the irreducible, separable, tracial, and density results.

Statement

Let C be the completed CAR algebra and Π its atomic representation on H_{at} . Use the source-defined family of homogeneity automorphisms and shell data to choose shell-link unitaries L_i on H_{at} . Define $A = C^*(\Pi(C) \cup \{L_i : i \in I\}) \subseteq B(H_{\text{at}})$, where $C^*(\dots)$ means the norm-closed unital $*$ -algebra generated by the displayed operators.

Definition

The Hilbert space H_{at} is the Hilbert direct sum of the selected pure-state GNS spaces, including the designated root component. The operators L_i are the full shell-link unitaries obtained from the shell data, not the individual partial-isometry elements of C . The algebra is generated inside an existing bounded-operator algebra; no universal completion is postulated.

Assumptions

The CAR algebra C , the distinguished root pure state, the selected atomic GNS representation Π on H_{at} , and the one homogeneity-based shell family have already been fixed by the construction. There is no additional assumption of simplicity, capture of representations, or the KOS theorem.

Conclusion

The abbreviation denotes the concrete closed unital $*$ -subalgebra A of $B(H_{\text{at}})$. Its carrier is used as a C^* -algebra in later declarations. This declaration defines A ; it does not by itself assert all the later counterexample properties.

Lean realization notes

Source kind: abbrev. Exact name: AtomicCounterexampleAlgebra. The elaborated value has type $\text{StarSubalgebra } \mathbb{C} B(\text{H}_{\text{at}})$; its use as a carrier employs the standard coercion. The chosen family is `homogeneityShellFamily` and the target is `ShellFamilyTarget` of that family. “Fixed” refers to one source-defined classical choice, not uniqueness of all possible presentations. The counterexample properties are established by the counterexample theorem for A; they are not inputs to this abbreviation.

Language: en · CARD_CONTENT_COMPLETE

Main citations

[MathlibAnnex.CStarAlgebra.CAR.homogeneityShellFamily](#)

[MathlibAnnex.CStarAlgebra.CAR.ShellFamilyTarget](#)

Lean source declaration (exact)

```
-- The single concrete C-star algebra used by every field of `AtomicCounterexampleEndpoint`.  
-/  
abbrev AtomicCounterexampleAlgebra := ShellFamilyTarget homogeneityShellFamily
```

Read exact [MathlibAnnex v0.4.0](#) source

Exact Card identity

Stable Card ID: a1397f366783063d30db5ab13e31d3e5339a0d31518fb16b965d5299478b8142

Card SHA-256: ba2e8276346975fcb757ba309080b548e3825fd7a9e5f505c0947c847694f385

Approved exposition SHA-256: a5bfc1e3ffa72a83a9fc18fc5eff9287d254f1f6c457d91462ad12de6cb4c896

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.