

The C*-algebra generated by the CAR representation and the shell unitaries

MathlibAnnex.CStarAlgebra.CAR.AtomicCounterexampleAlgebra

abbrev

Defines the single algebra T used in the irreducible and separable faithful representations.

Statement

Let A be the completed CAR algebra and let ω be its distinguished pure state. Let I be the set of unitary-equivalence classes of pure-state GNS representations. Choose a representative $\pi_i : A \rightarrow B(H_i)$ in each class, keeping the GNS representation of ω in its distinguished class, and put $\mathcal{H} = \bigoplus_{i \in I} H_i$ and $\pi = \bigoplus_{i \in I} \pi_i$. Fix the automorphisms and shell partial isometries supplied by CAR homogeneity, and choose the corresponding unitaries L_i on $\mathcal{H}$ by the cited shell construction. Define

$$T = C^*(\pi(A) \cup \{L_i : i \in I\}) \subseteq B(\mathcal{H}).$$

Definition

T is the norm-closed unital complex $*$ -subalgebra of $B(\mathcal{H})$ generated by $\pi(A)$ and the unitaries L_i . The automorphisms, shell partial isometries and resulting unitaries are chosen once; the abbreviation does not choose another isomorphic copy of the algebra.

Assumptions

The CAR algebra, the pure state ω , the GNS representatives and the unitaries L_i are those in the cited fixed construction. The index set I is not assumed countable.

Conclusion

The notation T refers to this particular generated C*-algebra. It does not denote a new isomorphic copy when different representations of T are considered.

Main citations

- Definition and its exact construction · Exact source
- The once-chosen representative shell family · Exact source
- The generated algebra for a fixed shell family · Exact source
- The fixed ambient link operators · Exact source

Lean source signature (exact)

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abbrev AtomicCounterexampleAlgebra := ShellFamilyTarget homogeneityShellFamily
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AtomicCounterexampleAlgebra is T , and homogeneityShellFamily is the fixed family of automorphisms and shell partial isometries. The full right-hand side is displayed. The separately cited ShellFamilyTarget and shellFamilyLinks explain its generated-algebra meaning; their definitions are not fragments of this abbreviation.

Lean realization notes

The abbreviation defines the algebra; separate theorems establish its simplicity, uniqueness of the nonzero irreducible representation up to unitary equivalence, and failure to be isomorphic to an algebra of compact operators. The fixed construction uses the proved CAR homogeneity theorem, without an additional KOS or CH hypothesis.

▼ Exact Card identity

Language: en

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