

Full operator image in finite dimension

`MathlibAnnex.Analysis.CStarAlgebra.Representation.surjective_of_irreducible_of_finiteDimensional`

Shows that an irreducible representation on a nonzero finite-dimensional Hilbert space has full operator image.

Statement

Let A be a unital complex C^* -algebra, H a nonzero finite-dimensional complex Hilbert space, and π an irreducible unital representation of A on H . No singleton hypothesis or separability assumption on A is made. π is surjective onto all bounded complex-linear operators on H .

Assumptions

Let A be a unital complex C^* -algebra, H a nonzero finite-dimensional complex Hilbert space, and π an irreducible unital representation of A on H . No singleton hypothesis or separability assumption on A is made.

Conclusion

π is surjective onto all bounded complex-linear operators on H .

Proof idea

The source finds preimages separately for the self-adjoint real and imaginary parts of an arbitrary operator T using finite-dimensional irreducible interpolation. It combines those preimages as $a+ib$ and invokes $T=\Re T+i\Im T$.

Proof steps

1. Exact Lean statement: $\forall \{A : \text{Type } u\} \text{ [inst : CStarAlgebra } A] \text{ [inst_1 : PartialOrder } A] \text{ [StarOrderedRing } A] \{H : \text{Type } v\} \text{ [inst_3 : NormedAddCommGroup } H] \text{ [inst_4 : InnerProductSpace } \mathbb{C} \ H] \text{ [inst_5 : CompleteSpace } H] \text{ [Nontrivial } H] \text{ [FiniteDimensional } \mathbb{C} \ H] (pi : \text{MathlibAnnex.Analysis.CStarAlgebra.Representation } A \ H), pi.\text{IsIrreducible} \rightarrow \text{Function.Surjective } \uparrow pi$
2. Surjective $\pi : A \rightarrow (H \rightarrow L[\mathbb{C}] \ H)$.
3. The source finds preimages separately for the self-adjoint real and imaginary parts of an arbitrary operator T using finite-dimensional irreducible interpolation. It combines those preimages as $a+ib$ and invokes $T=\Re T+i\Im T$.

Main citations

Lean source signature (exact)

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theorem surjective_of_irreducible_of_finiteDimensional [Nontrivial H]
[FiniteDimensional  $\mathbb{C}$  H]
(pi : Representation A H) (hirr : pi.IsIrreducible) :
Function.Surjective pi
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[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: a6b19f4652225ea9cada7c02f05c27707d0bf713859210f56ebbbf0df233cd00

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AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source-exposition correspondence, not an independent proof audit.