

Nonunital irreducibility

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.IsIrreducible`

Defines nonunital irreducibility as nonzero action with no nontrivial closed reducing subspaces.

Statement

Let A be a complex C^* -algebra, not assumed unital, H a complex Hilbert space, and π a representation of A on H . The predicate says that π acts nontrivially and every closed complex submodule L invariant under both $\pi(a)$ and its adjoint for every a is either $\{0\}$ or H .

Definition

$\text{IsIrreducible } \pi \leftrightarrow (\exists a : A, \pi a \neq 0) \wedge \forall L, \text{Reduces } \pi L \rightarrow L = \perp \vee L = \top.$

Assumptions

Let A be a complex C^* -algebra, not assumed unital, H a complex Hilbert space, and π a representation of A on H .

Conclusion

The predicate says that π acts nontrivially and every closed complex submodule L invariant under both $\pi(a)$ and its adjoint for every a is either $\{0\}$ or H .

Main citations

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation`

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

```
def IsIrreducible (pi : NonUnitalCStarRepresentation A H) : Prop
```

Read exact [MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: `c6768fb635e387f90ed7c53fa90f5b4dfebcf526a42da7af360fbfdefc2284b6`

Card SHA-256: `bc9645084bd34c210153e9a1ceb61fa2f603b776edf5ab668678f9356e023b96`

Approved exposition SHA-256: `7c05ef49afa4760ab5b5c89dca86bc234c51c0cde643c5c9628c471972e25eb1`

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