Maximal abelian subalgebra containing a self-adjoint element

`MathlibAnnex.Analysis.CStarAlgebra.exists_maximalAbelian_containing_isSelfAdjoint`

Places a self-adjoint element inside a maximal abelian unital star subalgebra.

Statement

Let x be self-adjoint in a unital complex C*-algebra A. There exists a maximal abelian unital star subalgebra D of A containing x.

Assumptions

Let x be self-adjoint in a unital complex C*-algebra A.

Conclusion

There exists a maximal abelian unital star subalgebra D of A containing x.

Proof idea

The source turns self-adjointness into IsStarNormal x and applies exists_maximalAbelian_containing. Normality is an intermediate premise, not the theorem's conclusion.

Proof steps

1. Exact Lean statement: ∀ {A : Type u} [inst : CStarAlgebra A] (x : A), IsSelfAdjoint x → ∃ D, MathlibAnnex.Analysis.CStarAlgebra.IsMaximalAbelian D ∧ x ∈ D

2. IsSelfAdjoint x → ∃ D : StarSubalgebra ℂ A, IsMaximalAbelian D ∧ x ∈ D.

3. The source turns self-adjointness into IsStarNormal x and applies exists_maximalAbelian_containing. Normality is an intermediate premise, not the theorem's conclusion.

Main citations

MathlibAnnex.Analysis.CStarAlgebra.IsMaximalAbelian

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

```
theorem exists_maximalAbelian_containing_isSelfAdjoint
(x : A) (hx : IsSelfAdjoint x) :
∃ D : StarSubalgebra ℂ A, IsMaximalAbelian D ∧ x ∈ D
```

Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: d4a5a6c0e99f5d6af2aa8ac954e42542f163c769257b313f68d69c5f1c0993e5

Card SHA-256: 3b716c28ce2e856618761c6501f718c76cf15bc712d79bcac94e6f1d09befae4

Approved exposition SHA-256: 8db4b0319fc9796edff8d40938630d9078cbcdafc89af56217b9f8ab24bbda1e

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.