Faithfulness of the separable tracial representation

`MathlibAnnex.CStarAlgebra.CAR.separableCounterexampleRepresentation_injective`

Proves injectivity of the specified representation rather than introducing a new algebra or a new witness.

Statement

The tracial representation $\rho: A \to B(H\tau)$ of the fixed CAR-based algebra on the CAR trace-GNS space is faithful.

Assumptions

Let $A \subseteq B(H_{at})$ be the fixed unital C*-algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C, and then taking the norm-closed unital *-algebra they generate. Let $H\tau = L^2(C, \tau C)$ be the GNS Hilbert space of the normalized CAR trace τC. Let ρ be the specified representation obtained by pointed unitary transport of the tracial model.

Conclusion

ρ is injective: for all $a, b \in A$, $\rho(a) = \rho(b)$ implies $a = b$. In particular, $\rho(a) = 0$ implies $a = 0$.

Proof route

Specialize injectivity of the family-parametric trace-model representation to the fixed homogeneity family.

Proof steps

1. Take the established injectivity theorem for the trace model of a representative shell family.
2. Apply it to the same family used to define A and ρ.

Lean realization notes

The literal conclusion is Function.Injective separableCounterexampleRepresentation. Separability of Hτ is a separate established fact; the isometry property and the faithfulness of the scalar trace are separate statements as well. Injectivity does not imply irreducibility here.

Language: en · CARD_CONTENT_COMPLETE

Main citations

MathlibAnnex.CStarAlgebra.CAR.traceModelRepresentation_injective

Lean source declaration (exact)

```
theorem separableCounterexampleRepresentation_injective :
Function.Injective separableCounterexampleRepresentation :=
traceModelRepresentation_injective homogeneityShellFamily
```

Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: d56473ecdc9e94ee50a17bbf8196f8a40160186b88f4c3a14e76126565a08c09

Card SHA-256: 82dcb53f14784d8184619f6b558bae0a063457b36614f838624b46b2451cb4c8

Approved exposition SHA-256: ffb7cff0b0cc32111ef358a7a8bebb19042f0c3a70e2ab2c54d971abaa5a60c4

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.