The norm density of A is exactly the continuum

`MathlibAnnex.CStarAlgebra.CAR.hasDensityCharacter_atomicCounterexampleAlgebra`

Proves both existence of a dense set of the stated size and minimality of that size.

Statement

The fixed CAR-based C*-algebra A has norm density character exactly $\mathfrak{c} = 2^{\aleph_0}$.

Assumptions

Let $A \subseteq B(H_{at})$ be the fixed unital C*-algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C, and then taking the norm-closed unital *-algebra they generate. Density is computed in the norm topology on A; no CH assumption is made.

Conclusion

There exists a norm-dense subset $D \subseteq A$ with $|D| = \mathfrak{c}$, and every norm-dense subset $E \subseteq A$ satisfies $\mathfrak{c} \leq |E|$. Thus dens(A) = $\mathfrak{c}$.

Proof route

Combine the carrier-cardinality upper bound with the lower bound for every norm-dense subset.

Proof steps

1. The full carrier A provides a dense witness of cardinality at most $\mathfrak{c}$.
2. Apply the lower bound for all norm-dense subsets of A to that witness to obtain exact cardinality $\mathfrak{c}$.
3. Apply the lower bound for all norm-dense subsets of A to an arbitrary dense subset for the universal minimality bound.

Lean realization notes

HasDensityCharacter contains an attained dense witness and a universal lower bound. This is not a statement about the strong-operator topology on a represented copy of A or an algebraic dimension.

Language: en · CARD_CONTENT_COMPLETE

Main citations

MathlibAnnex.CStarAlgebra.CAR.continuum_le_cardinalMk_dense_atomicCounterexampleAlgebra

MathlibAnnex.Topology.HasDensityCharacter.of_cardinalMk_le_of_forall_dense

Lean source declaration (exact)

```
/-- The exact norm density of the same fixed algebra is continuum. -/
theorem hasDensityCharacter_atomicCounterexampleAlgebra :
HasDensityCharacter AtomicCounterexampleAlgebra Cardinal.continuum :=
HasDensityCharacter.of_cardinalMk_le_of_forall_dense
cardinalMk_atomicCounterexampleAlgebra_le_continuum
continuum_le_cardinalMk_dense_atomicCounterexampleAlgebra
```

Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: e184de0db0422d3780170324fd831e1a89afcbecc672ebf5b54159a4f9478b4f

Card SHA-256: bbaa38f0690d7b0ca134e7875ae37f1e48d6f2cd8dc0d5b1d6b4c3701fa214c4

Approved exposition SHA-256: 20091753c8207919644eda72991caa1705f87a7ece308bf93d912f1fdf702907

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.