The atomic inclusion representation

`MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleRepresentation`

Defines the representation on the atomic Hilbert space, keeping it distinct from the separable tracial model.

Statement

Define $\pi_{at}: A \to B(H_{at})$ to be the inclusion representation of the fixed CAR-based operator algebra A. Thus $\pi_{at}(a)x = a(x)$.

Definition

The map is literal subtype inclusion. Its injectivity follows from this construction; its irreducibility is a further theorem, recorded in the counterexample theorem for A, rather than part of the meaning of a representation. H_{at} is not the CAR trace-GNS space $H\tau$.

Assumptions

Let $A \subseteq B(H_{at})$ be the fixed unital C*-algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C, and then taking the norm-closed unital *-algebra they generate. Regard A as a closed unital *-subalgebra of $B(H_{at})$, with its inherited algebraic operations.

Conclusion

A specified unital complex *-algebra homomorphism $\pi_{at}: A \to B(H_{at})$, given by the underlying operator of each element of A.

Lean realization notes

Source kind: noncomputable def. Exact name: atomicCounterexampleRepresentation. Representation A H is the unital *-homomorphism API. This definition specializes shellFamilyInclusion to homogeneityShellFamily.

Language: en · CARD_CONTENT_COMPLETE

Main citations

MathlibAnnex.CStarAlgebra.CAR.shellFamilyInclusion

Lean source declaration (exact)

```
/-- The fixed faithful irreducible displayed representation of `AtomicCounterexampleAlgebra`.
-/
noncomputable def atomicCounterexampleRepresentation :
Representation AtomicCounterexampleAlgebra
(MathlibAnnex.CStarAlgebra.PureState.SelectedAtomicHilbert completedRootPureState) :=
shellFamilyInclusion homogeneityShellFamily
```

Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: e4cb90a633cf73821fb0608918e5b50c598cc2a23fc76340409b027afff8c9fa

Card SHA-256: 516a57a8f062ef0a6234d796e52b96c61eb3b141eb155771b24f65817dddb0ea

Approved exposition SHA-256: b2608b0b12cb6115777cb5f05c41771d130f931c3459ef8fb554c55f7b520b87

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source–exposition correspondence, not an independent proof audit.