The ambient representation of the fixed algebra

`MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleRepresentation`

def

Defines the inclusion representation $\rho_0 : T \to B(\mathcal{H})$.

Statement

Let A be the completed CAR algebra and $T = C^*(\pi(A) \cup \{L_i : i \in I\}) \subseteq B(\mathcal{H})$ the algebra generated by the selected direct sum π of pure-state GNS representations indexed by their unitary-equivalence classes $i \in I$ and the unitaries L_i from the fixed homogeneity and shell construction. Write $\iota(a) = \pi(a)$ for the embedding $A \to T$. The inclusion representation is

$$\rho_0 : T \longrightarrow B(\mathcal{H}), \qquad \rho_0(t)x = t(x).$$

Definition

The map ρ_0 is the literal inclusion: $\rho_0(t) = t$ as a bounded operator on $\mathcal{H}$. The source map $\iota(a)$ is $\pi(a)$ regarded as an element of T, so $\rho_0 \circ \iota = \pi$.

Assumptions

The GNS direct sum, the unitaries L_i and the generated algebra T are fixed as above. Each $t \in T$ is already a bounded operator on $\mathcal{H}$.

Conclusion

ρ_0 is a unital $*$-representation and $\rho_0 \circ \iota = \pi$. Its definition introduces no new choice of Hilbert space or operators.

Main citations

- Definition and its exact construction · Exact source
- The inclusion before specialization · Exact source
- The CAR embedding into the same generated algebra · Exact source
- Faithfulness of the inclusion · Exact source
- Irreducibility of the inclusion · Exact source

Lean source signature (exact)

```
noncomputable def atomicCounterexampleRepresentation :
    Representation AtomicCounterexampleAlgebra
      (MathlibAnnex.CStarAlgebra.PureState.SelectedAtomicHilbert completedRootPureState) :=
  shellFamilyInclusion homogeneityShellFamily
```

> `atomicCounterexampleRepresentation` is ρ_0. `SelectedAtomicHilbert completedRootPureState` is $\mathcal{H}$; `completedRootPureState` is ω bundled with its purity proof. The right-hand side specializes `shellFamilyInclusion` to the same family. The map ι is explained through the separately cited source-map declaration.

Lean realization notes

Faithfulness and irreducibility of this particular inclusion are separate theorems cited below. They are not additional data chosen in the definition; no separability of $\mathcal{H}$ is assumed.

▼ **Exact Card identity**

Language: en
CARD_CONTENT_COMPLETE
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Card revision: 2 · SHA-256: 516a57a8f062ef0a6234d796e52b96c61eb3b141eb155771b24f65817dddb0ea
Exposition revision: 3 · SHA-256: cef5d882030e1a2f04b20a8daa51af0a5c0030a977e83892fd660334d8f908ea
Source commit: 437e6e46228bbb8e91211ebded349d7a30020e73
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