Center transport under bounded point-reflection symmetry

`MathlibAnnex.IsometryEquiv.map_center_of_mapsTo_pointReflection`

Identifies the distinguished centers of two bounded reflection-invariant subsets under an isometry equivalence.

Statement

Let $f : s \simeq_i t$ be an isometry equivalence between subsets of real normed affine spaces. Suppose $c \in s$ and $d \in t$, the source set s is bounded, and s and t are invariant under point reflection about c and d respectively. Then $f(c)=d$ as subtype points.

Assumptions

Real normed affine spaces; source and target sets contain their centers and are invariant under reflection through those centers; the source is bounded; f is an isometry equivalence.

Conclusion

f maps the source reflection center to the target reflection center.

Proof idea

Conjugate the target reflection about d by f to obtain a self-isometry g of s. Every self-isometry of the bounded source reflection space fixes c. Translating that fixed-point equation back through f shows that f(c) is fixed by reflection about d, hence equals d.

Proof steps

1. Restrict target point reflection about d to an isometry equivalence of t.
2. Conjugate this restricted reflection by f to form a self-isometry g of s.
3. Apply the bounded reflection-center fixed-point lemma to g at c.
4. Transport the resulting equality through f to show that restricted reflection fixes f(c).
5. Forget the subtype and apply the characterization of fixed points of point reflection to conclude $f(c)=d$.

Main citations

_private.MathlibAnnex.Analysis.Normed.Affine.Reflection.0.MathlibAnnex.IsometryEquiv.apply_center_eq_of_isBounded_of_mapsTo_pointReflection

The proof or construction of Center transport under bounded point-reflection symmetry uses the project declaration “Every self-isometry fixes the center of a bounded reflection-invariant set” at the indicated step.

_private.MathlibAnnex.Analysis.Normed.Affine.Reflection.0.MathlibAnnex.IsometryEquiv.restrictedPointReflection

The proof or construction of Center transport under bounded point-reflection symmetry uses the project declaration “Point reflection restricted to an invariant subset” at the indicated step.

Lean source signature (exact)

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theorem map_center_of_mapsTo_pointReflection
{s : Set P} {t : Set Q} {c : P} {d : Q} (f : s ≃ᵢ t)
(hc : c ∈ s) (hd : d ∈ t) (hs : IsBounded s)
(hsreflect : MapsTo (pointReflection ℝ c) s s)
(htreflect : MapsTo (pointReflection ℝ d) t t) :
f ⟨c, hc⟩ = ⟨d, hd⟩
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Read exact MathlibAnnex v0.4.0 source

Exact Card identity

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