

The representation on the CAR trace-GNS space

`MathlibAnnex.CStarAlgebra.CAR.separableCounterexampleRepresentation`

Defines a separable-space representation of exactly the same algebra A .

Statement

Define $\rho: A \rightarrow B(H_\tau)$ on the CAR trace-GNS space $H_\tau = L^2(C, \tau_C)$ by transporting the previously constructed tracial representation along the pointed unitary identifying its CAR-cyclic model with H_τ .

Definition

The unitary V preserves the distinguished cyclic trace vector and intertwines the CAR actions. Conjugating the tracial representation by V places the target algebra on the original CAR trace-GNS space. Separability of H_τ and faithfulness of ρ are established separately; this definition does not assert irreducibility.

Assumptions

Let $A \subseteq B(H_{at})$ be the fixed unital C^* -algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C , and then taking the norm-closed unital $*$ -algebra they generate. Let $H_\tau = L^2(C, \tau_C)$ be the GNS Hilbert space of the normalized CAR trace τ_C . Use the fixed homogeneity shell family and the chosen pointed unitary $V: H_\tau \rightarrow H_{tr}$ from the CAR GNS space to the constructed target-state GNS space.

Conclusion

A specified unital complex $*$ -homomorphism $\rho: A \rightarrow B(H_\tau)$, with $\rho(a) = V^{-1} \pi_{tr}(a) V$. The domain is the same A as in the atomic representation.

Lean realization notes

Source kind: noncomputable def. Exact name: separableCounterexampleRepresentation. SeparableCounterexampleHilbertSpace abbreviates TraceHilbertSpace. The definition is traceModelRepresentation homogeneityShellFamily. The faithfulness theorem for the separable tracial representation proves injectivity; the exclusion of separable irreducible representations excludes nonzero irreducibility on a separable space.

Language: en · CARD_CONTENT_COMPLETE

Main citations

[MathlibAnnex.CStarAlgebra.CAR.traceModelRepresentation](#)

Lean source declaration (exact)

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-- A faithful representation of exactly the existing atomic counterexample
on a separable Hilbert space. -/
noncomputable def separableCounterexampleRepresentation :
Representation AtomicCounterexampleAlgebra SeparableCounterexampleHilbertSpace :=
traceModelRepresentation homogeneityShellFamily
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[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: f0336a753a8f202333b869aaaeb1d7630c9a50e9edb8c7ec3a05b58fb89ab611

Card SHA-256: 47d4f0b7c34f13bd52725a8e51e896bc85168489e7892a0a473cf0eac009e425

Approved exposition SHA-256: 18a00b16c602643f4721dea28608dc386a4a9424793aec633962350320bd0ef4

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source-exposition correspondence, not an independent proof audit.