

No separably acting representative of all nonzero irreducible representations

`MathlibAnnex.Analysis.CStarAlgebra.Representation.not_singleton_amongNonUnital_of_infiniteDimensional`

Shows that, for an infinite-dimensional unital C^* -algebra, no representation on a separable Hilbert space is a representative of the unique unitary-equivalence class of all nonzero irreducible representations.

Statement

Let A be a nonzero infinite-dimensional unital complex C^* -algebra, H a separable complex Hilbert space, and π a unital representation of A on H . The representation π is not a representative of the unique unitary-equivalence class of nonzero irreducible representations of A . Equivalently, π is not both irreducible and unitarily equivalent to every nonzero irreducible representation of A . This does not rule out separable irreducible representations in general.

Assumptions

Let A be a nonzero infinite-dimensional unital complex C^* -algebra, H a separable complex Hilbert space, and π a unital representation of A on H .

Conclusion

The representation π is not a representative of the unique unitary-equivalence class of nonzero irreducible representations of A . Equivalently, π is not both irreducible and unitarily equivalent to every nonzero irreducible representation of A . This does not rule out separable irreducible representations in general.

Proof idea

If π represented the unique irreducible-representation class of A , the preceding finite-dimensional-algebra theorem would force A to be finite-dimensional, contradicting the hypothesis.

Proof steps

1. Exact Lean statement: $\forall \{A : \text{Type } u\} \text{ [inst : CStarAlgebra } A] \text{ [inst_1 : PartialOrder } A] \text{ [StarOrderedRing } A] \{H : \text{Type } v\} \text{ [inst_3 : NormedAddCommGroup } H] \text{ [inst_4 : InnerProductSpace } \mathbb{C} \ H] \text{ [inst_5 : CompleteSpace } H] \text{ [Nontrivial } A], \neg \text{FiniteDimensional } \mathbb{C} \ A \rightarrow \forall \text{ [TopologicalSpace.SeparableSpace } H] (\pi : \text{MathlibAnnex.Analysis.CStarAlgebra.Representation } A \ H), \neg \pi.\text{IsSingletonIrreducibleModelAmongNonUnital}$
2. $\neg \text{FiniteDimensional } \mathbb{C} \ A \rightarrow \forall \pi \text{ on separable } H, \neg \pi.\text{IsSingletonIrreducibleModelAmongNonUnital}$
3. If π represented the unique irreducible-representation class of A , the preceding finite-dimensional-algebra theorem would force A to be finite-dimensional, contradicting the hypothesis.

Main citations

`MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional_algebra_of_singleton_amongNonUnital`

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

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theorem not_singleton_amongNonUnital_of_infiniteDimensional
[Nontrivial A] (hA : ¬ FiniteDimensional ℂ A)
[TopologicalSpace.SeparableSpace H]
(pi : Representation A H) :
¬ IsSingletonIrreducibleModelAmongNonUnital.{u, v, u} pi
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[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: f2686115c0a5647dbadc2b23862bf82db33c3680599f938b1b6342daaf7d4467

Card SHA-256: f21cea9af3dce8c0c987722a576294b1d5d1053d1a5473e2956074cae45cb850

Approved exposition SHA-256: 9ad261346687a42639eda4fa4932baee2e699894c2622fa730c4569b19b276f7

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