

Finite-dimensional algebra from a separable representative of the unique irreducible-representation class

`MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional_algebra_of_singleton_amongNonUnital`

Shows that a nonzero unital C^* -algebra is finite-dimensional when it has a representative of its unique irreducible-representation class on a separable Hilbert space.

Statement

Let A be a nonzero unital complex C^* -algebra, H a separable complex Hilbert space, and π a unital representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A . Then A is finite-dimensional over $\mathbb{C}$; finite dimension of A is the conclusion, not an initial hypothesis.

Assumptions

Let A be a nonzero unital complex C^* -algebra, H a separable complex Hilbert space, and π a unital representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A .

Conclusion

A itself is finite-dimensional over $\mathbb{C}$; finite dimension of A is the conclusion, not an initial hypothesis.

Proof idea

The source converts the quantifier and applies the unital algebra-finiteness theorem. The underlying argument first makes H finite-dimensional and π injective, then embeds A into the finite-dimensional bounded-operator space on H .

Proof steps

1. Exact Lean statement: $\forall \{A : \text{Type } u\} [\text{inst} : \text{CStarAlgebra } A] [\text{inst}_1 : \text{PartialOrder } A] [\text{StarOrderedRing } A] \{H : \text{Type } v\} [\text{inst}_3 : \text{NormedAddCommGroup } H] [\text{inst}_4 : \text{InnerProductSpace } \mathbb{C} H] [\text{inst}_5 : \text{CompleteSpace } H] [\text{Nontrivial } A] [\text{TopologicalSpace.SeparableSpace } H] (\pi : \text{MathlibAnnex.Analysis.CStarAlgebra.Representation } A H), \pi.\text{IsSingletonIrreducibleModelAmongNonUnital} \rightarrow \text{FiniteDimensional } \mathbb{C} A$
2. `FiniteDimensional  $\mathbb{C} A$ .`
3. The source converts the quantifier and applies the unital algebra-finiteness theorem. The underlying argument first makes H finite-dimensional and π injective, then embeds A into the finite-dimensional bounded-operator space on H .

Main citations

`MathlibAnnex.Analysis.CStarAlgebra.Representation.IsSingletonIrreducibleModelAmongNonUnital`

Exact formal dependency; inspect the linked Card and exact source.

`MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional_space_of_singleton`

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

```
theorem finiteDimensional_algebra_of_singleton_amongNonUnital
[Nontrivial A] [TopologicalSpace.SeparableSpace H]
(pi : Representation A H)
(hsingle : IsSingletonIrreducibleModelAmongNonUnital.{u, v, u} pi) :
FiniteDimensional  $\mathbb{C}$  A
```

[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: f34adabc1cac70dceb32acb212d5f7c0b85c259c2c330087932a6f3fe9f5e8a9

Card SHA-256: ecc79438064f62aae0bbacb681b74e61a1bdf6907db81f63936cc28abaaa4b2d

Approved exposition SHA-256: 3910c52590590990ffd018d67dee9f0baf1b08920998b22a4402c0781b9282db

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source-exposition correspondence, not an independent proof audit.