

The complete ordinary counterexample assertion

`MathlibAnnex.CStarAlgebra.CAR.AtomicCounterexampleEndpoint`

Names the precise collection of properties later established for the fixed algebra.

Statement

For the fixed CAR-based algebra A , its CAR map $j: C \rightarrow A$, and its atomic inclusion $\pi_{\text{at}}: A \rightarrow B(H_{\text{at}})$, define the ordinary counterexample assertion to be the conjunction of the properties listed below.

Definition

A is nontrivial and norm closed in $B(H_{\text{at}})$.

The CAR source map $j: C \rightarrow A$ is injective and satisfies $j(1) = 1$.

A is infinite-dimensional over $\mathbb{C}$.

The inclusion $\pi_{\text{at}}: A \rightarrow B(H_{\text{at}})$ is faithful, nonzero, and irreducible.

For every complex Hilbert space K and every nonzero irreducible $*$ -representation $\sigma: A \rightarrow B(K)$, not initially required to preserve the unit, there is a surjective complex-linear isometry $U: H_{\text{at}} \rightarrow K$ with $U(\pi_{\text{at}}(a)x) = \sigma(a)(Ux)$ for all $a \in A$ and $x \in H_{\text{at}}$.

Every norm-closed two-sided ideal of A is either $\{0\}$ or A .

For every complex Hilbert space K , there is no injective complex-linear $*$ -homomorphism $e: A \rightarrow B(K)$, not required to preserve the unit, whose image consists of compact operators and contains every compact operator on K .

Assumptions

Let $A \subseteq B(H_{\text{at}})$ be the fixed unital C^* -algebra obtained by adjoining the chosen shell-link unitaries to the atomic representation of the CAR algebra C , and then taking the norm-closed unital $*$ -algebra they generate. Fix the source map j and the ambient inclusion π_{at} from that same construction. Comparison spaces K are arbitrary complete complex Hilbert spaces in the independent comparison universe; no separability is required.

Conclusion

The result of this abbreviation is a proposition consisting of the listed ten source fields. Defining that proposition is distinct from proving it; the counterexample theorem for A supplies the proof.

Lean realization notes

Source kind: abbrev with result Prop. Exact name: AtomicCounterexampleEndpoint. The seven displayed groups preserve all ten fields of ShellFamilyEndpoint: nontrivial_target, isClosed_target, source_injective, source_unital, not_finiteDimensional_target, ambient_injective, isIrreducible_ambient, captures_nonunital, closedIdeal_dichotomy, not_compactOperatorModel. Here irreducibility includes nonzeroness. The last field is an exclusion of a model of the entire compact-operator algebra, not of one particular embedding. No separable, trace, cardinality, or density condition is part of this definition.

Language: en · CARD_CONTENT_COMPLETE

Main citations

MathlibAnnex.CStarAlgebra.CAR.ShellFamilyEndpoint

Lean source declaration (exact)

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-- Fully expanded ordinary endpoint for the single target chosen above. -/  
abbrev AtomicCounterexampleEndpoint : Prop :=  
ShellFamilyEndpoint.{v} homogeneityShellFamily
```

[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: fc53edababae4cd5e177061b0866789eacc62e1d9a69cf1dc16c08c02d6307a

Card SHA-256: 0e0dcb2302827d125d7dbd94b5cd58de45152fb64dc29f81686b47253b995036

Approved exposition SHA-256: 181e50c5834e6c5b8704381fcef3561447c84893a9555e5f24a5eaae4c19b14d

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