

Finite-dimensional space for a separable representative of the unique irreducible-representation class

`MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional_space_of_singleton_amongNonUnital`

Shows that the Hilbert space of a separably acting representative of the unique irreducible-representation class is finite-dimensional.

Statement

Let A be a nonzero unital complex C^* -algebra, H a separable complex Hilbert space, and π a unital representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A . Then H is finite-dimensional over $\mathbb{C}$.

Assumptions

Let A be a nonzero unital complex C^* -algebra, H a separable complex Hilbert space, and π a unital representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A .

Conclusion

H is finite-dimensional over $\mathbb{C}$.

Proof idea

The source passes from the nonunital-interface formulation of the unique-class condition to its unital formulation and applies the finite-dimensional-space theorem, whose compact-identity route yields the result.

Proof steps

1. Exact Lean statement: $\forall \{A : \text{Type } u\} \text{ [inst : CStarAlgebra } A] \text{ [inst_1 : PartialOrder } A] \text{ [StarOrderedRing } A] \{H : \text{Type } v\} \text{ [inst_3 : NormedAddCommGroup } H] \text{ [inst_4 : InnerProductSpace } \mathbb{C} \ H] \text{ [inst_5 : CompleteSpace } H] \text{ [Nontrivial } A] \text{ [TopologicalSpace.SeparableSpace } H] \text{ (pi : MathlibAnnex.Analysis.CStarAlgebra.Representation } A \ H), \text{ pi.IsSingletonIrreducibleModelAmongNonUnital} \rightarrow \text{FiniteDimensional } \mathbb{C} \ H$
2. `FiniteDimensional  $\mathbb{C}$  H`.
3. The source passes from the nonunital-interface formulation of the unique-class condition to its unital formulation and applies the finite-dimensional-space theorem, whose compact-identity route yields the result.

Main citations

`MathlibAnnex.Analysis.CStarAlgebra.Representation.IsSingletonIrreducibleModelAmongNonUnital`

Exact formal dependency; inspect the linked Card and exact source.

`MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional_space_of_singleton`

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

```
theorem finiteDimensional_space_of_singleton_amongNonUnital
[Nontrivial A] [TopologicalSpace.SeparableSpace H]
(pi : Representation A H)
(hsingle : IsSingletonIrreducibleModelAmongNonUnital.{u, v, u} pi) :
FiniteDimensional  $\mathbb{C}$  H
```

[Read exact MathlibAnnex v0.4.0 source](#)

Exact Card identity

Stable Card ID: fd18e1a684c11c6d461ab0afb2c4c47e3636fb5b84f9a7cca39b472897e0f86d

Card SHA-256: 6c39a0335bae06c780f4d6169369e5075367719237ff4ec7c2428f09aff37038

Approved exposition SHA-256: db4d9902c74fe8e2aa0a6c08c108dbb7619311863e6dff597280d01c3acfec3

AI-assisted material. Selected, edited, and reviewed by Ryotaro Tanaka. The recorded review is source-exposition correspondence, not an independent proof audit.