Countability of the full character space

`MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.countable_characterSpace_of_nonUnital_singleton`

Shows that the full character space of a closed unitization subalgebra is countable when π is a separably acting representative of the unique irreducible-representation class.

Statement

Let A be a nonzero complex C*-algebra, not assumed unital; let H be a separable complex Hilbert space; let π be a representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A; and let D be a closed unital star subalgebra of Unitization ℂ A. The entire character space of D is countable, including the distinguished scalar character.

Assumptions

Let A be a nonzero complex C*-algebra, not assumed unital; let H be a separable complex Hilbert space; let π be a representation of A on H that is a representative of the unique unitary-equivalence class of nonzero irreducible representations of A; and let D be a closed unital star subalgebra of Unitization ℂ A.

Conclusion

The entire character space of D is countable, including the distinguished scalar character.

Proof idea

For each character other than infinityCharacterOn D, the prior source theorem supplies a joint unit eigenvector. Distinct characters give orthogonal vectors, so separability of H makes that complement countable. The source then maps the disjoint sum of the complement and one Unit onto the entire character space.

Proof steps

1. Exact Lean statement: ∀ {A : Type u} [inst : NonUnitalCStarAlgebra A] [inst_1 : PartialOrder A] [StarOrderedRing A] {H : Type v} [inst_3 : NormedAddCommGroup H] [inst_4 : InnerProductSpace ℂ H] [inst_5 : CompleteSpace H] [Nontrivial A] [TopologicalSpace.SeparableSpace H] (pi : MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation A H), pi.IsSingletonIrreducibleModel → ∀ (D : StarSubalgebra ℂ (Unitization ℂ A)) [inst_8 : IsClosed ↑D], Countable ↑(WeakDual.characterSpace ℂ ↥D)

2. Countable (WeakDual.characterSpace ℂ D).

3. For each character other than infinityCharacterOn D, the prior source theorem supplies a joint unit eigenvector. Distinct characters give orthogonal vectors, so separability of H makes that complement countable. The source then maps the disjoint sum of the complement and one Unit onto the entire character space.

Main citations

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists_unit_eigenvector_of_character_ne_infinity

Exact formal dependency; inspect the linked Card and exact source.

Lean source signature (exact)

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theorem countable_characterSpace_of_nonUnital_singleton [Nontrivial A]
[TopologicalSpace.SeparableSpace H]
(pi : NonUnitalCStarRepresentation A H)
(hsingle : IsSingletonIrreducibleModel.{u, v, u} pi)
(D : StarSubalgebra ℂ (Unitization ℂ A))
[IsClosed (D : Set (Unitization ℂ A))] :
Countable (WeakDual.characterSpace ℂ D)
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Read exact MathlibAnnex v0.4.0 source

Exact Card identity

Stable Card ID: ff5886cf6420336dbe578275731ad0361f521340e5e9760ec098cd284273b584

Card SHA-256: 22237a1d60bd1b4dae08a2f0dfe2cc4ccdc6f96f85b6be11b110839d3e1671f9

Approved exposition SHA-256: 7b776aba3a7c6f586aa065a772ec1bce99bedc1fe5993759ef7c1987de61bc69

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