Mankiewicz Extension Theorem

Theorem Project | Presentation R1

The eleven declaration explanations have been reviewed for correspondence with their exact Lean source. This is not a line-by-line human audit of the proof scripts. Public Declaration Card links are not active.

Principal result

An isometry equivalence from an open connected subset onto an open subset of real normed spaces extends uniquely to an affine isometry equivalence of the ambient spaces. The selected route also reaches convex-domain and ball specializations.

Exact source

MathlibAnnex v0.2.0 · Project entry · Project Source Manifest

Commit: 30963f26ac8ffa3dc3e9ec9de91fd0f9daf05305

Tree: b3378851a5287f5e3c8418f668e4d0ba52718739

11 declaration explanations; 52 reachability pairs; 10 display edges; maximum level 8. Boundary Inputs: 961 external or omitted-support dependencies, outside the selected level nodes.

Dependency-first declaration route

Level 0 - Center transport under bounded point-reflection symmetry

Declaration: MathlibAnnex.IsometryEquiv.map_center_of_reflectionInvariant

Let $f:s\simeq_i t$ be an isometry equivalence between subsets of real normed affine spaces. Suppose $c\in s$ and $d\in t$, the source set s is bounded, and s and t are invariant under point reflection about c and d respectively. Then $f(c)=d$ as subtype points.

Assumptions: Real normed affine spaces; source and target sets contain their centers and are invariant under reflection through those centers; the source is bounded; f is an isometry equivalence.

Conclusion: f maps the source reflection center to the target reflection center.

Exact source

Level 0 - Symmetric lens

Declaration: MathlibAnnex.symmetricLens

`symmetricLens x y r` is the intersection of the two closed balls of radius r centered at x and y.

Assumptions: A normed additive commutative group E; points x and y; a real radius r.

Conclusion: The symmetric lens is the intersection of the closed radius-r balls centered at x and y. This definition needs no positivity assumption.

Exact source

Level 1 - Midpoint preservation on a symmetric lens

Declaration: MathlibAnnex.IsometryEquiv.map_midpoint_of_symmetricLens

Assume the midpoint of x,y belongs to symmetricLens(x,y,r), and likewise the midpoint of x',y' belongs to symmetricLens(x',y',r), as expressed by the two half-distance inequalities. Any isometry equivalence between these lenses maps midpoint(x,y) to midpoint(x',y').

Assumptions: Real normed vector spaces; an isometry equivalence between symmetric lenses of the same radius r; both half-distances between the respective foci are at most r.

Conclusion: The isometry maps the midpoint of the source foci to the midpoint of the target foci.

Exact source

Level 2 - Midpoint preservation under lens containment

Declaration: MathlibAnnex.IsometryEquiv.map_midpoint_of_symmetricLens_subset

Let f:s≃ᵢt and x,y∈s. If the symmetric lens of radius r around x,y lies in s, the corresponding lens around f(x),f(y) lies in t, and half the distance between x and y is at most r, then f sends midpoint(x,y) to midpoint(f(x),f(y)).

Assumptions: Real normed vector spaces; f is an isometry equivalence s to t; radius-r symmetric lenses about x,y and f(x),f(y) lie in s and t; half the distance from x to y is at most r.

Conclusion: f preserves the midpoint of x and y. The target half-distance bound follows from the isometry.

Exact source

Level 3 - Midpoint preservation on a quarter ball

Declaration: MathlibAnnex.IsometryEquiv.map_midpoint_of_mem_ball

Let f:s≃ᵢt and let c∈s. Assume R>0, ball(c,R)⊆s, and ball(f(c),R)⊆t. If x,y∈s both lie in ball(c,R/4), then f sends their midpoint to the midpoint of f(x) and f(y).

Assumptions: Real normed vector spaces; radius-R balls about c and f(c) lie in the source and target; R > 0; x,y lie in the radius-R/4 source ball.

Conclusion: f sends the midpoint of x and y to the midpoint of f(x) and f(y).

Exact source

Level 4 - Affine-segment preservation on a quarter ball

Declaration: MathlibAnnex.IsometryEquiv.map_lineMap_of_mem_ball

Under the same ambient-ball hypotheses as the local midpoint theorem, if x and y lie in ball(c,R/4), then for every a in [0,1], f sends the affine point lineMap(x,y,a) to lineMap(f(x),f(y),a).

Assumptions: Real normed vector spaces; radius-R balls about c and f(c) lie in the source and target; R > 0; x,y lie in the radius-R/4 source ball; a lies in [0,1].

Conclusion: f preserves the affine combination (1-a)x + ay, with the exact subtype membership witnesses supplied in Lean.

Exact source

Level 5 - Local affine-isometry chart on a smaller ball

Declaration: MathlibAnnex.IsometryEquiv.exists_affineExtension_eqOn_ball

Suppose a set isometry f:s≃ᵢt is defined on subsets containing the ambient balls ball(c,R) and ball(f(c),R), with R>0. Then there is an ambient real affine isometry equivalence A that agrees with f at every source point lying in the smaller ball ball(c,R/8).

Assumptions: Real normed vector spaces; f is an isometry equivalence s to t; R > 0; radius-R balls about c and f(c) are contained in s and t.

Conclusion: An ambient affine isometry equivalence agrees with f on the ball about c of radius R/8.

Exact source

Level 6 - Mankiewicz extension on open connected domains

Declaration: MathlibAnnex.IsometryEquiv.existsUnique_affineExtension

Let f be a surjective isometry from an open connected subset s of a real normed space onto an open subset t of another real normed space. Then there exists a unique ambient real affine isometry equivalence A agreeing with f on all of s. Target connectedness is not a separate assumption.

Assumptions: Real normed vector spaces E and F; s is open and connected; t is open; f is a surjective isometry from s to t.

Conclusion: There is a unique ambient real affine isometry equivalence whose restriction to s is f.

Exact source

Level 7 - Affine extension from open balls

Declaration: MathlibAnnex.IsometryEquiv.existsUnique_affineExtension_ball

Let f be a surjective isometry from the open ball ball(c,r) onto the open ball ball(d,R) in real normed spaces, where r>0 and R>0; the two radii need not be equal. Then there is a unique ambient real affine isometry equivalence A such that A(x)=f(x) for every x in the source ball.

Assumptions: Real normed vector spaces; f is an isometry equivalence between open balls with radii r > 0 and R > 0.

Conclusion: f extends uniquely to an ambient real affine isometry equivalence. The two positive radii need not be assumed equal.

Exact source

Level 7 - Convex-set extension via ambient interiors

Declaration: MathlibAnnex.IsometryEquiv.existsUnique_affineExtension_of_convex

Let f be a surjective isometry from a convex subset s of a real normed space onto a subset t. Assume the ambient interior of s is nonempty and, for every x in s, x lies in interior(s) if and only if f(x) lies in interior(t). Then f extends uniquely to an ambient real affine isometry equivalence. No separate convexity assumption on t is required by this statement.

Assumptions: Real normed vector spaces; the source s is convex with nonempty interior; f is an isometry equivalence s to t; x is interior to s exactly when f(x) is interior to t.

Conclusion: f extends uniquely to an ambient real affine isometry equivalence. No additional convexity assumption on t is stated.

Exact source

Level 8 - Affine extension from equal-radius closed balls

Declaration: MathlibAnnex.IsometryEquiv.existsUnique_affineExtension_closedBall

Let f be a surjective isometry between the closed balls closedBall(c,r) and closedBall(d,r) of the same radius r>0 in real normed spaces. Then f has a unique ambient real affine isometry-equivalence extension agreeing with f on the whole source closed ball.

Assumptions: Real normed vector spaces; f is an isometry equivalence between closed balls with the same radius r > 0.

Conclusion: f extends uniquely to an ambient real affine isometry equivalence.

Exact source

Technical source and review details

Source-exposition review: OWNER_REVIEWED_SOURCE_EXPOSITION_CORRESPONDENCE. This concerns the correspondence of the explanations to exact source, not a line-by-line human audit of proof scripts.

Source qualification run: 35086615666. Source, graph, levels, Boundary Inputs and review evidence are inherited without re-execution. The level labels above are read directly from the bound level map.

Availability and publication history of this Project view are recorded by the hosting release. Public Declaration Cards: 0; canonical Card links: 0.