

CAR foundations and canonical states

The finite matrix stages $A_n = M_{2^n}(\mathbb{C})$ enter their norm completion A through ι_n . The root state ω and normalized trace τ have different roles: $\omega(\iota_n(a)) = a_{00}$, whereas $\tau(\iota_n(a)) = 2^{-n} \text{Tr}(a)$. The root projections are $f_n = \iota_n(E_{00})$. Here E_{00} belongs to A_n and $f_n = e_{00}^{(n)}$ is its image in A ; that image need not have rank one in a representation of A .

The compression estimate $\|f_n x f_n - \omega(x) f_n\| \rightarrow 0$ is pointwise in the fixed element x . It supplies the later residual-space arguments. The simplicity proof uses a sufficiently late fixed projection; it does not replace this norm estimate by uniform convergence. Infinite dimension and the absence of nonzero compact images in irreducible representations are separate conclusions. Their proofs remain in the individual Cards.

Exact Card references

- The normalized trace on a finite CAR stage — `MathlibAnnex.CStarAlgebra.CAR.stageTrace`
- The CAR algebra as a completion of finite matrix stages — `MathlibAnnex.CStarAlgebra.CAR.Limit`
- The product-vector state on the completed CAR algebra — `MathlibAnnex.CStarAlgebra.CAR.rootState`
- Purity of the completed root state — `MathlibAnnex.CStarAlgebra.CAR.isPureState_rootState`
- The completed CAR algebra is infinite-dimensional — `MathlibAnnex.CStarAlgebra.CAR.not_finiteDimensional`
- A finite-row average centralizes its matrix stage — `MathlibAnnex.CStarAlgebra.CAR.ofStage_commute_rowAverage`
- Constructing the normalized trace on the completed CAR algebra — `MathlibAnnex.CStarAlgebra.CAR.trace`
- Normalization and the trace identity determine the CAR trace — `MathlibAnnex.CStarAlgebra.CAR.eq_trace_of_apply_one_of_mul_comm`
- The decreasing root projections in the CAR completion — `MathlibAnnex.CStarAlgebra.CAR.rootFlag`
- Root compression becomes scalar in norm — `MathlibAnnex.CStarAlgebra.CAR.tendsto_norm_compressionError`
- Simplicity of the completed CAR algebra — `MathlibAnnex.CStarAlgebra.CAR.isSimpleCStarAlgebra_limit`
- An irreducible CAR representation has no nonzero compact image — `MathlibAnnex.CStarAlgebra.CAR.eq_zero_of_isCompactOperator_image`

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

Exact source and provider boundary · Earlier 466-declaration Project view and PDF

Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

The normalized trace on a finite CAR stage

`MathlibAnnex.CStarAlgebra.CAR.stageTrace`

The normalized matrix trace provides a positive tracial functional compatible with the CAR inclusions.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Constructing the normalized trace on the completed CAR algebra

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LEVEL 0

The CAR algebra as a completion of finite matrix stages

`MathlibAnnex.CStarAlgebra.CAR.Limit`

Names the completed CAR algebra in which finite matrix calculations can be extended by norm approximation.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

The product-vector state on the completed CAR algebra, A finite-row average centralizes its matrix stage, The decreasing root projections in the CAR completion, Constructing the normalized trace on the completed CAR algebra, The completed CAR algebra is infinite-dimensional

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Level 1

LEVEL 1

The product-vector state on the completed CAR algebra

`MathlibAnnex.CStarAlgebra.CAR.rootState`

Compatible evaluation at the distinguished matrix coordinate extends continuously to the CAR completion.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The CAR algebra as a completion of finite matrix stages

USED BY IN THIS PROJECT

Root compression becomes scalar in norm, Purity of the completed root state

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LEVEL 1

The completed CAR algebra is infinite-dimensional

`MathlibAnnex.CStarAlgebra.CAR.not_finiteDimensional`

Rules out finite dimension by retaining matrix subalgebras of unbounded dimension.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The CAR algebra as a completion of finite matrix stages

USED BY IN THIS PROJECT

An irreducible CAR representation has no nonzero compact image

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LEVEL 1

A finite-row average centralizes its matrix stage

`MathlibAnnex.CStarAlgebra.CAR.ofStage_commute_rowAverage`

Turns an arbitrary element of the completed CAR algebra into one commuting with a prescribed finite matrix stage.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The CAR algebra as a completion of finite matrix stages

USED BY IN THIS PROJECT

None in this scope

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LEVEL 1

Constructing the normalized trace on the completed CAR algebra

MathlibAnnex.CStarAlgebra.CAR.trace

Carries compatible bounded matrix traces through the algebraic limit, normed union and completion.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The normalized trace on a finite CAR stage, The CAR algebra as a completion of finite matrix stages

USED BY IN THIS PROJECT

Normalization and the trace identity determine the CAR trace

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LEVEL 1

The decreasing root projections in the CAR completion

MathlibAnnex.CStarAlgebra.CAR.rootFlag

The distinguished stage corners give a nested sequence of projections supporting the root state.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The CAR algebra as a completion of finite matrix stages

USED BY IN THIS PROJECT

Root compression becomes scalar in norm

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Level 2

LEVEL 2

Purity of the completed root state

`MathlibAnnex.CStarAlgebra.CAR.isPureState_rootState`

Passes extremality of the root vector states from every finite matrix stage to the completed CAR algebra.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The product-vector state on the completed CAR algebra

USED BY IN THIS PROJECT

None in this scope

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LEVEL 2

Normalization and the trace identity determine the CAR trace

`MathlibAnnex.CStarAlgebra.CAR.eq_trace_of_apply_one_of_mul_comm`

Proves uniqueness without assuming positivity of the competing continuous functional.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Constructing the normalized trace on the completed CAR algebra

USED BY IN THIS PROJECT

None in this scope

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LEVEL 2

Root compression becomes scalar in norm

`MathlibAnnex.CStarAlgebra.CAR.tendsto_norm_compressionError`

Extends an exact rank-one compression identity on finite stages to a pointwise norm limit on the completed algebra.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The product-vector state on the completed CAR algebra, The decreasing root projections in the CAR completion

USED BY IN THIS PROJECT

Simplicity of the completed CAR algebra

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Level 3

LEVEL 3

Simplicity of the completed CAR algebra

`MathlibAnnex.CStarAlgebra.CAR.isSimpleCStarAlgebra_limit`

Every nonzero closed two-sided ideal of the CAR algebra contains the identity.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Root compression becomes scalar in norm

USED BY IN THIS PROJECT

An irreducible CAR representation has no nonzero compact image

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Level 4

LEVEL 4

An irreducible CAR representation has no nonzero compact image

`MathlibAnnex.CStarAlgebra.CAR.eq_zero_of_isCompactOperator_image`

Simplicity and infinite dimensionality rule out compact operators coming from nonzero CAR elements.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Simplicity of the completed CAR algebra, The completed CAR algebra is infinite-dimensional

USED BY IN THIS PROJECT

None in this scope

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