

Finite-stage purification and local transport

A represented root corner has enough Hilbert-space dimension to realize any finite Gram matrix. Matrix-unit reconstruction then purifies a vector state on a selected finite stage. Supported self-adjoint interpolation and corner amplification turn finite vector transport into unitary paths; the approximate state tests concern every matrix entry, not only diagonal entries.

The protected-set theorem chooses its stage and tolerance before the later state-approximation request. The unrestricted theorem instead purifies on a large stage and transports at stage zero, where the sole unit-vector test is automatic. These are different stages. The fixed-stage abbreviation $e = e_{00}^{(n)}$, the finite-dimensional interpolation space E , and the amplification $L_n(z)$ are local objects, not the later shell links L_i or trace-GNS unitary E .

Exact Card references

- Realizing a finite Gram matrix in a represented CAR corner —
MathlibAnnex.CStarAlgebra.CAR.exists_rootCornerFamily_with_gram
- Purifying a vector state on a finite CAR stage —
MathlibAnnex.CStarAlgebra.CAR.exists_unitVector_vectorFunctional_eq_on_stage
- Lifting a corner involution to an ambient CAR unitary —
MathlibAnnex.CStarAlgebra.CAR.exists_liftedCornerExponential_apply_eq_involution
- Exact vector transport along an almost central unitary path —
MathlibAnnex.CStarAlgebra.CAR.exists_stageTests_exact_unitary_path_apply_eq_and_conjugate_sub_norm_It
- Cross-representation state approximation with a protected finite set —
MathlibAnnex.CStarAlgebra.CAR.exists_stageTests_crossRepresentation_path_approx
- Approximating another vector state along a unitary path —
MathlibAnnex.CStarAlgebra.CAR.exists_unitary_crossRepresentation_path_approx

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

Exact source and provider boundary · Earlier 466-declaration Project view and PDF

Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

Realizing a finite Gram matrix in a represented CAR corner

`MathlibAnnex.CStarAlgebra.CAR.exists_rootCornerFamily_with_gram`

The infinite-dimensional root corner contains an isometric copy of any finite family of vectors.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Purifying a vector state on a finite CAR stage

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LEVEL 0

Lifting a corner involution to an ambient CAR unitary

`MathlibAnnex.CStarAlgebra.CAR.exists_liftedCornerExponential_apply_eq_involution`

A prescribed unitary involution on a represented root corner can be realized on finitely many vectors by one lifted corner exponential.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Exact vector transport along an almost central unitary path, Approximating another vector state along a unitary path

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Level 1

LEVEL 1

Purifying a vector state on a finite CAR stage

`MathlibAnnex.CStarAlgebra.CAR.exists_unitVector_vectorFunctional_eq_on_stage`

Every finite-stage restriction of a unit vector state can be realized by a unit vector in any irreducible CAR representation.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Realizing a finite Gram matrix in a represented CAR corner

USED BY IN THIS PROJECT

Approximating another vector state along a unitary path, Cross-representation state approximation with a protected finite set

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LEVEL 1

Exact vector transport along an almost central unitary path

`MathlibAnnex.CStarAlgebra.CAR.exists_stageTests_exact_unitary_path_apply_eq_and_conjugate_sub_norm_lt`

Finite matrix-state tests ensure exact transport while protecting a prescribed finite set throughout the path.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Lifting a corner involution to an ambient CAR unitary

USED BY IN THIS PROJECT

Cross-representation state approximation with a protected finite set

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Level 2

LEVEL 2

Cross-representation state approximation with a protected finite set

`MathlibAnnex.CStarAlgebra.CAR.exists_stageTests_crossRepresentation_path_approx`

Finite entrywise tests allow new state approximation while an entire unitary path nearly fixes earlier data.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Purifying a vector state on a finite CAR stage, Exact vector transport along an almost central unitary path

USED BY IN THIS PROJECT

None in this scope

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LEVEL 2

Approximating another vector state along a unitary path

`MathlibAnnex.CStarAlgebra.CAR.exists_unitary_crossRepresentation_path_approx`

A fixed irreducible CAR representation can approximate any vector state on finitely many elements by moving a given unit vector.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Purifying a vector state on a finite CAR stage, Lifting a corner involution to an ambient CAR unitary

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None in this scope

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