

Pure-state homogeneity and shell matching

For pure states φ, ψ on CAR, the sequence construction controls both $\alpha_n(a)$ and the actual inverses $\alpha_n^{-1}(a)$. The limiting automorphism satisfies $\varphi \circ \alpha = \psi$. The detailed alternating correction inequalities, the protected inverse images, and the substitution $R_n^{-1}(d_j)$ are retained in the key proof Card; this route does not duplicate that proof.

For one fixed approximately inner automorphism, shell matching gives $w_{i,n}^* w_{i,n} = \alpha_i(p_n)$ and $w_{i,n} w_{i,n}^* = p_n$, where $p_n = f_n - f_{n+1}$. At the root class $o = [\omega]$, the chosen family uses $\alpha_o = \text{id}$ and $w_{o,n} = p_n$. Each other component uses one automorphism for all of its shells. The generic projection-matching theorem is not a new proof of the general KOS proposition. The same chosen homogeneity family is used in the construction of the algebra and in the classification of its irreducible representations.

Exact Card references

- Approximately inner homogeneity of pure CAR states —
MathlibAnnex.CStarAlgebra.CAR.PureStateHomogeneity
- A two-sided inner intertwining sequence for two states —
MathlibAnnex.CStarAlgebra.CAR.HasInnerIntertwiningSequence
- Two-sided inner sequences imply pure-state homogeneity —
MathlibAnnex.CStarAlgebra.CAR.homogeneity_of_innerIntertwiningSequences
- Pure CAR states admit two-sided inner intertwining sequences —
MathlibAnnex.CStarAlgebra.CAR.hasInnerIntertwiningSequence_of_pure
- Pure-state homogeneity of the completed CAR algebra — MathlibAnnex.CStarAlgebra.CAR.homogeneity
- Exact projection links for one approximately inner automorphism —
MathlibAnnex.CStarAlgebra.exists_shell_family_of_approximately_inner
- A fixed representative family of CAR shell data —
MathlibAnnex.CStarAlgebra.CAR.RepresentativeShellFamily
- Choosing the CAR shell family from proved homogeneity —
MathlibAnnex.CStarAlgebra.CAR.homogeneityShellFamily

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

Exact source and provider boundary · Earlier 466-declaration Project view and PDF

Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

Approximately inner homogeneity of pure CAR states

`MathlibAnnex.CStarAlgebra.CAR.PureStateHomogeneity`

The homogeneity property asks for exact state transport by an automorphism that is locally approximable by inner automorphisms.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Two-sided inner sequences imply pure-state homogeneity

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LEVEL 0

A two-sided inner intertwining sequence for two states

`MathlibAnnex.CStarAlgebra.CAR.HasInnerIntertwiningSequence`

An inner sequence records both forward and actual inverse convergence data before any limiting automorphism is constructed.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Two-sided inner sequences imply pure-state homogeneity, Pure CAR states admit two-sided inner intertwining sequences

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LEVEL 0

Exact projection links for one approximately inner automorphism

`MathlibAnnex.CStarAlgebra.exists_shell_family_of_approximately_inner`

A point-norm approximation followed by close-projection conjugacy gives exact support identities for an arbitrary projection family.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Choosing the CAR shell family from proved homogeneity

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LEVEL 0

A fixed representative family of CAR shell data

`MathlibAnnex.CStarAlgebra.CAR.RepresentativeShellFamily`

One family records a transporting automorphism and exact shell links for each selected pure-state class, with an explicitly fixed root component.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Choosing the CAR shell family from proved homogeneity

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Level 1

LEVEL 1

Two-sided inner sequences imply pure-state homogeneity

`MathlibAnnex.CStarAlgebra.CAR.homogeneity_of_innerIntertwiningSequences`

The forward and inverse limits close the homogeneity argument once an intertwining sequence is supplied for every pure-state pair.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Approximately inner homogeneity of pure CAR states, A two-sided inner intertwining sequence for two states

USED BY IN THIS PROJECT

Pure-state homogeneity of the completed CAR algebra

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LEVEL 1

Pure CAR states admit two-sided inner intertwining sequences

`MathlibAnnex.CStarAlgebra.CAR.hasInnerIntertwiningSequence_of_pure`

Finite-set vector-state transport supplies explicit corrections; their summable conjugation bounds and state estimates yield the required two-sided sequence.

IMMEDIATE PREREQUISITES IN THIS PROJECT

A two-sided inner intertwining sequence for two states

USED BY IN THIS PROJECT

Pure-state homogeneity of the completed CAR algebra

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Level 2

LEVEL 2

Pure-state homogeneity of the completed CAR algebra

`MathlibAnnex.CStarAlgebra.CAR.homogeneity`

The constructed inner intertwining sequences discharge the supplier hypothesis and give an approximately inner transporting automorphism.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Two-sided inner sequences imply pure-state homogeneity, Pure CAR states admit two-sided inner intertwining sequences

USED BY IN THIS PROJECT

Choosing the CAR shell family from proved homogeneity

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Level 3

LEVEL 3

Choosing the CAR shell family from proved homogeneity

`MathlibAnnex.CStarAlgebra.CAR.homogeneityShellFamily`

CAR homogeneity and exact shell matching supply one family whose distinguished component is fixed by an explicit identity choice.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Exact projection links for one approximately inner automorphism, A fixed representative family of CAR shell data, Pure-state homogeneity of the completed CAR algebra

USED BY IN THIS PROJECT

None in this scope

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