Direct sums of GNS representations and shell unitaries

Let I denote pure-GNS equivalence classes and (H_i, π_i, ξ_i) the root-preserving selected data. Distinct classes, not merely distinct state functionals, give inequivalent representations. Their direct sum is π on $\mathcal{H} = \bigoplus_i H_i$. Write $e_i : H_i \to \mathcal{H}$ and $b_i = e_i \xi_i$.

The generic atomic-shell theorem takes the rank-one limiting defects as hypotheses. The pure-GNS specialization discharges irreducibility, inequivalence and normalization, while retaining the shell hypotheses. The separate CAR realization proves the required residual-line identities. It chooses unitary links once. The generic generated algebra permits arbitrary bounded extra operators and does not itself assert the CAR consequences.

Exact Card references

- The selected GNS representation of a pure-state class — MathlibAnnex.CStarAlgebra.PureState.selectedRepresentation
- Distinct GNS classes have no unitary intertwiner — MathlibAnnex.CStarAlgebra.PureState.no_unitaryIntertwiner_selectedRepresentation
- The atomic direct sum of the selected CAR representations — MathlibAnnex.CStarAlgebra.CAR.selectedAtomicRepresentation
- An irreducible atomic model from projection shells — MathlibAnnex.CStarAlgebra.AtomicConstruction.exists_irreducible_atomicShellModel
- The atomic-shell construction on selected pure-GNS fibers — MathlibAnnex.CStarAlgebra.AtomicConstruction.exists_irreducible_pureAtomicShellModel
- The closed operator algebra generated by a representation and extra operators — MathlibAnnex.CStarAlgebra.AtomicConstruction.concreteTarget
- Realizing a shell family by a faithful irreducible operator algebra — MathlibAnnex.CStarAlgebra.CAR.exists_completedAtomicShellModel

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

Exact source and provider boundary · Earlier 466-declaration Project view and PDF

Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

The selected GNS representation of a pure-state class

`MathlibAnnex.CStarAlgebra.PureState.selectedRepresentation`

A choice of representative pure states, fixed literally at a root, gives one concrete GNS representation per equivalence class.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Distinct GNS classes have no unitary intertwiner, The atomic direct sum of the selected CAR representations

Open Declaration Card **Read exact source**

LEVEL 0

The closed operator algebra generated by a representation and extra operators

`MathlibAnnex.CStarAlgebra.AtomicConstruction.concreteTarget`

Places a represented algebra and a chosen family of bounded operators in one concrete closed algebra.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

An irreducible operator algebra constructed from projection shells

Open Declaration Card **Read exact source**

Level 1

LEVEL 1

Distinct GNS classes have no unitary intertwiner

`MathlibAnnex.CStarAlgebra.PureState.no_unitaryIntertwiner_selectedRepresentation`

The class index makes the selected pure-GNS family pairwise unitarily inequivalent.

IMMEDIATE PREREQUISITES IN THIS PROJECT
The selected GNS representation of a pure-state class

USED BY IN THIS PROJECT
The atomic-shell construction on selected pure-GNS fibers

Open Declaration Card **Read exact source**

LEVEL 1

The atomic direct sum of the selected CAR representations

`MathlibAnnex.CStarAlgebra.CAR.selectedAtomicRepresentation`

All selected pure-GNS fibers of the completed CAR algebra act together on one Hilbert direct sum.

IMMEDIATE PREREQUISITES IN THIS PROJECT
The selected GNS representation of a pure-state class

USED BY IN THIS PROJECT
Realizing a shell family by a faithful irreducible operator algebra

Open Declaration Card **Read exact source**

LEVEL 1

An irreducible operator algebra constructed from projection shells

`MathlibAnnex.CStarAlgebra.AtomicConstruction.exists_irreducible_atomicShellModel`

Shell partial isometries with rank-one limiting defects can be completed to unitary links that join inequivalent irreducible fibers.

IMMEDIATE PREREQUISITES IN THIS PROJECT
The closed operator algebra generated by a representation and extra operators

USED BY IN THIS PROJECT
The atomic-shell construction on selected pure-GNS fibers

Open Declaration Card **Read exact source**

Level 2

LEVEL 2

The atomic-shell construction on selected pure-GNS fibers

`MathlibAnnex.CStarAlgebra.AtomicConstruction.exists_irreducible_pureAtomicShellModel`

Pure-state GNS data supplies the irreducible and inequivalent fibers required by the generic shell construction.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Distinct GNS classes have no unitary intertwiner, An irreducible operator algebra constructed from projection shells

USED BY IN THIS PROJECT

Realizing a shell family by a faithful irreducible operator algebra

Open Declaration Card **Read exact source**

Level 3

LEVEL 3

Realizing a shell family by a faithful irreducible operator algebra

`MathlibAnnex.CStarAlgebra.CAR.exists_completedAtomicShellModel`

Constructs unitary links between pure-state summands and an irreducible algebra containing a faithful copy of CAR.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The atomic direct sum of the selected CAR representations, The atomic-shell construction on selected pure-GNS fibers

USED BY IN THIS PROJECT

None in this scope

Open Declaration Card **Read exact source**