Unitary equivalence of irreducible representations

For a fixed supplied family and its once-chosen links, put $T = C^*(\pi(A), \{L_i\})$. The maps have distinct domains:

$$\iota : A \to T, \qquad \rho_0 : T \to B(\mathcal{H}), \qquad \rho_0 \circ \iota = \pi.$$

The selected vector states give a cyclic-sum isometry $W : \mathcal{H} \to K$. In the generated-algebra setting, root normalization and irreducibility show that the chosen isometry is surjective. The full equivalence proof uses that same underlying map after bundling it as a unitary, and additionally uses the prescribed cyclic-vector action of the links. Shell sums are compared separately on the two Hilbert spaces; strong continuity of an arbitrary representation is never assumed.

The nonunital version first proves unit preservation for the same nonzero irreducible representation. Simplicity and compact-operator exclusion are subsequent consequences. The existential unitary in a separately applied theorem need not equal a witness selected in another application.

Exact Card references

- The CAR source map into the fixed shell-family target — MathlibAnnex.CStarAlgebra.CAR.shellFamilySourceHom
- The ambient inclusion of the fixed shell-family target — MathlibAnnex.CStarAlgebra.CAR.shellFamilyInclusion
- Assembling selected GNS cyclic subspaces into an isometric source representation — MathlibAnnex.CStarAlgebra.CAR.exists_selectedAtomicCyclicIsometry
- The cyclic sum fills every irreducible target representation — MathlibAnnex.CStarAlgebra.CAR.exists_surjective_selectedAtomicCyclicIsometry
- Every irreducible target representation is the displayed operator model — MathlibAnnex.CStarAlgebra.CAR.ambientInclusion_unitaryEquivalent
- Irreducible representations are captured without assuming unitality — MathlibAnnex.CStarAlgebra.CAR.shellFamilyInclusion_unitaryEquivalent_nonUnital
- Closed-ideal simplicity of the shell-generated algebra — MathlibAnnex.CStarAlgebra.CAR.isSimpleCStarAlgebra_shellFamilyTarget
- The shell-generated algebra is not an algebra of all compact operators — MathlibAnnex.CStarAlgebra.CAR.not_isCompactOperatorModel_shellFamilyTarget
- One fixed shell-generated algebra and its representation-theoretic endpoint — MathlibAnnex.CStarAlgebra.CAR.shellFamilyEndpoint

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

Exact source and provider boundary · Earlier 466-declaration Project view and PDF

Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

The CAR source map into the fixed shell-family target

`MathlibAnnex.CStarAlgebra.CAR.shellFamilySourceHom`

Restricting the codomain of the selected atomic representation gives the source map into one fixed generated target.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

The shell-generated algebra is not an algebra of all compact operators, Closed-ideal simplicity of the shell-generated algebra

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LEVEL 0

The ambient inclusion of the fixed shell-family target

`MathlibAnnex.CStarAlgebra.CAR.shellFamilyInclusion`

The same concrete target has a literal representation on the selected atomic Hilbert space.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

Closed-ideal simplicity of the shell-generated algebra, Unitary equivalence without an initial unitality assumption

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LEVEL 0

Assembling selected GNS cyclic subspaces into an isometric source representation

`MathlibAnnex.CStarAlgebra.CAR.exists_selectedAtomicCyclicIsometry`

Joins mutually orthogonal pure-state cyclic copies and controls the limiting flag projections on their sum.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

The cyclic sum fills every irreducible target representation

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Level 1

LEVEL 1

The cyclic sum fills every irreducible target representation

`MathlibAnnex.CStarAlgebra.CAR.exists_surjective_selectedAtomicCyclicIsometry`

Uses shell reconstruction and residual lines to turn an isometric CAR intertwiner into a unitary equivalence on the source.

IMMEDIATE PREREQUISITES IN THIS PROJECT
Assembling selected GNS cyclic subspaces into an isometric source representation

USED BY IN THIS PROJECT
Every irreducible representation is unitarily equivalent to the inclusion

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LEVEL 1

The shell-generated algebra is not an algebra of all compact operators

`MathlibAnnex.CStarAlgebra.CAR.not_isCompactOperatorModel_shellFamilyTarget`

Excludes an injective representation of the infinite-dimensional unital algebra onto all compact operators.

IMMEDIATE PREREQUISITES IN THIS PROJECT
The CAR source map into the fixed shell-family target

USED BY IN THIS PROJECT
Structural properties of one shell-generated C*-algebra

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Level 2

LEVEL 2

Every irreducible representation is unitarily equivalent to the inclusion

`MathlibAnnex.CStarAlgebra.CAR.ambientInclusion_unitaryEquivalent`

Extends the unitary equivalence from the CAR source to all additional generators and the norm-closed target.

IMMEDIATE PREREQUISITES IN THIS PROJECT
The cyclic sum fills every irreducible target representation

USED BY IN THIS PROJECT
Closed-ideal simplicity of the shell-generated algebra, Unitary equivalence without an initial unitality assumption

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Level 3

LEVEL 3

Unitary equivalence without an initial unitality assumption

`MathlibAnnex.CStarAlgebra.CAR.shellFamilyInclusion_unitaryEquivalent_nonUnital`

Shows why allowing nonunital input representations does not enlarge the irreducible representation class of the shell-generated algebra.

IMMEDIATE PREREQUISITES IN THIS PROJECT
The ambient inclusion of the fixed shell-family target, Every irreducible representation is unitarily equivalent to the inclusion

USED BY IN THIS PROJECT
Structural properties of one shell-generated C*-algebra

Open Declaration Card **Read exact source**

LEVEL 3

Closed-ideal simplicity of the shell-generated algebra

`MathlibAnnex.CStarAlgebra.CAR.isSimpleCStarAlgebra_shellFamilyTarget`

Uses a faithful irreducible representation in the unique unitary-equivalence class to rule out proper nonzero closed two-sided ideals.

IMMEDIATE PREREQUISITES IN THIS PROJECT
The ambient inclusion of the fixed shell-family target, Every irreducible representation is unitarily equivalent to the inclusion, The CAR source map into the fixed shell-family target

USED BY IN THIS PROJECT
Structural properties of one shell-generated C*-algebra

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Level 4

LEVEL 4

Structural properties of one shell-generated C*-algebra

`MathlibAnnex.CStarAlgebra.CAR.shellFamilyEndpoint`

Collects the proved properties of a single generated algebra, keeping the shell-family input separate from its consequences.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The shell-generated algebra is not an algebra of all compact operators, Closed-ideal simplicity of the shell-generated algebra, Unitary equivalence without an initial unitality assumption

USED BY IN THIS PROJECT

None in this scope

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