

Trace extensions and faithful representations on a separable space

Fix the same T and ι , and first extend the CAR trace τ to a state ϕ on T . Existence precedes uniqueness and traciality. In each trace-realizing cyclic representation, the estimates for f_n and $\alpha_j(f_n)$ kill the residual projections on the source-cyclic subspace. Target cyclicity then proves source cyclicity. The fixed-left-hand-side limit calculation is retained with its full n dependence in the pointed-transport Card.

Pointed transport proves uniqueness among all state extensions. Uniqueness gives invariance under source unitaries; their span and the actual represented shell limits put all generators in the closed centralizer. This proves traciality. A different restriction argument then proves uniqueness among all tracial states.

The CAR trace space and the extension-state space are different: the selected unitary points as $E : H_\tau \rightarrow H_\phi$, and $\tilde{\rho}(t) = E^{-1}\rho_\phi(t)E$. This E is not the capture map W . Faithfulness of a representation is distinct from faithfulness of its vector state. The Hilbert spaces are separable here; norm separability of T is not assumed.

Exact Card references

- Extending the CAR trace to the fixed shell target —
MathlibAnnex.CStarAlgebra.CAR.exists_state_extension_trace
- Uniqueness among all state extensions of the CAR trace —
MathlibAnnex.CStarAlgebra.CAR.existsUnique_state_extension_trace
- The unique trace extension is tracial on the whole target —
MathlibAnnex.CStarAlgebra.CAR.traceExtension_mul_comm
- The fixed shell target has a unique tracial state —
MathlibAnnex.CStarAlgebra.CAR.existsUnique_tracial_state
- The GNS representation of the target trace extension —
MathlibAnnex.CStarAlgebra.CAR.tracialRepresentation
- Faithfulness of the target-state GNS representation —
MathlibAnnex.CStarAlgebra.CAR.tracialRepresentation_injective
- The target represented on the CAR trace GNS space —
MathlibAnnex.CStarAlgebra.CAR.traceModelRepresentation
- Faithfulness of the model on the CAR trace Hilbert space —
MathlibAnnex.CStarAlgebra.CAR.traceModelRepresentation_injective
- Pointed unitary transport between trace-cyclic target representations —
MathlibAnnex.CStarAlgebra.CAR.exists_pointed_unitary_of_trace_of_cyclic

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

Exact source and provider boundary · Earlier 466-declaration Project view and PDF

Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

Extending the CAR trace to the fixed shell target

`MathlibAnnex.CStarAlgebra.CAR.exists_state_extension_trace`

The source trace first becomes a state on the existing target algebra.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

The GNS representation of the target trace extension

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LEVEL 0

Pointed unitary transport between trace-cyclic target representations

`MathlibAnnex.CStarAlgebra.CAR.exists_pointed_unitary_of_trace_of_cyclic`

Source cyclicity and strong shell sums extend the canonical CAR orbit isometry to the whole target.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

The unique trace extension is tracial on the whole target, Uniqueness among all state extensions of the CAR trace

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Level 1

LEVEL 1

The GNS representation of the target trace extension

`MathlibAnnex.CStarAlgebra.CAR.tracialRepresentation`

The chosen state on the existing target supplies its cyclic GNS representation.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Extending the CAR trace to the fixed shell target

USED BY IN THIS PROJECT

The target represented on the CAR trace GNS space, Faithfulness of the target-state GNS representation, The unique trace extension is tracial on the whole target, Uniqueness among all state extensions of the CAR trace

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Level 2

LEVEL 2

Uniqueness among all state extensions of the CAR trace

`MathlibAnnex.CStarAlgebra.CAR.existsUnique_state_extension_trace`

Pointed transport identifies arbitrary state extensions before traciality is proved.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The GNS representation of the target trace extension, Pointed unitary transport between trace-cyclic target representations

USED BY IN THIS PROJECT

None in this scope

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LEVEL 2

The unique trace extension is tracial on the whole target

`MathlibAnnex.CStarAlgebra.CAR.traceExtension_mul_comm`

Unitary invariance, strong shell sums, and a closed centralizer extend the trace identity beyond the source.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The GNS representation of the target trace extension, Pointed unitary transport between trace-cyclic target representations

USED BY IN THIS PROJECT

The fixed shell target has a unique tracial state

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LEVEL 2

Faithfulness of the target-state GNS representation

MathlibAnnex.CStarAlgebra.CAR.tracialRepresentation_injective

The ideal dichotomy of the same target forces the constructed nonzero representation to be injective.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The GNS representation of the target trace extension

USED BY IN THIS PROJECT

Faithfulness of the representation on the CAR trace Hilbert space

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LEVEL 2

The target represented on the CAR trace GNS space

MathlibAnnex.CStarAlgebra.CAR.traceModelRepresentation

A fixed pointed unitary transports the target action onto the Hilbert space constructed from CAR alone.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The GNS representation of the target trace extension

USED BY IN THIS PROJECT

Faithfulness of the representation on the CAR trace Hilbert space

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Level 3

LEVEL 3

The fixed shell target has a unique tracial state

`MathlibAnnex.CStarAlgebra.CAR.existsUnique_tracial_state`

Every tracial state is forced back to the CAR trace and hence to its unique extension.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The unique trace extension is tracial on the whole target

USED BY IN THIS PROJECT

None in this scope

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LEVEL 3

Faithfulness of the representation on the CAR trace Hilbert space

`MathlibAnnex.CStarAlgebra.CAR.traceModelRepresentation_injective`

Unitary conjugation preserves injectivity of the target-state representation.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The target represented on the CAR trace GNS space, Faithfulness of the target-state GNS representation

USED BY IN THIS PROJECT

None in this scope

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