

One C*-algebra and its representation-theoretic properties

Specialize the supplied family to the once-chosen homogeneity family. The algebra remains $T = C^*(\pi(A), \{L_i\})$, with inclusion ρ_0 . The structural theorem gives its faithful irreducible inclusion and the unique nonzero irreducible representation class, including possibly nonunital inputs.

The same T acts faithfully on the separable CAR trace space by $\tilde{\rho}(t) = E^{-1}\rho_\phi(t)E$. It has no nonzero irreducible representation on a separable Hilbert space. The proof first establishes unit preservation and composes the two classification unitaries in the direction $U_\sigma U_\rho^{-1}$ before applying the finite-dimensionality theorem. This does not rule out its irreducible inclusion on the nonseparable atomic space.

Exact Card references

- The C*-algebra generated by the CAR representation and the shell unitaries —
MathlibAnnex.CStarAlgebra.CAR.AtomicCounterexampleAlgebra
- The ambient representation of the fixed algebra —
MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleRepresentation
- A simple C*-algebra with a unique irreducible representation class —
MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleEndpoint
- The fixed algebra on the CAR trace Hilbert space —
MathlibAnnex.CStarAlgebra.CAR.separableCounterexampleRepresentation
- Faithfulness of the representation on the CAR trace Hilbert space —
MathlibAnnex.CStarAlgebra.CAR.separableCounterexampleRepresentation_injective
- No nonzero irreducible representation on a separable Hilbert space —
MathlibAnnex.CStarAlgebra.CAR.not_isIrreducible_of_separable
- A separably represented C*-algebra with no separable irreducible representation —
MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleEndpoint_and_exists_separable_faithful_representation_and_no_separable_irreducible_representation

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

Exact source and provider boundary · Earlier 466-declaration Project view and PDF

Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

The C*-algebra generated by the CAR representation and the shell unitaries

MathlibAnnex.CStarAlgebra.CAR.AtomicCounterexampleAlgebra

Defines the single algebra T used in the irreducible and separable faithful representations.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

The fixed algebra on the CAR trace Hilbert space, The ambient representation of the fixed algebra

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LEVEL 0

A simple C*-algebra with a unique irreducible representation class

MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleEndpoint

Establishes the properties that make the fixed algebra T a counterexample to Naimark's problem.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

USED BY IN THIS PROJECT

No nonzero irreducible representation on a separable Hilbert space

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Level 1

LEVEL 1

The ambient representation of the fixed algebra

`MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleRepresentation`

Defines the inclusion representation $\rho_0 : T \rightarrow B(\mathcal{H})$.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The C*-algebra generated by the CAR representation and the shell unitaries

USED BY IN THIS PROJECT

No nonzero irreducible representation on a separable Hilbert space

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LEVEL 1

The fixed algebra on the CAR trace Hilbert space

`MathlibAnnex.CStarAlgebra.CAR.separableCounterexampleRepresentation`

Defines a representation of the same algebra T on the GNS Hilbert space of the CAR trace.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The C*-algebra generated by the CAR representation and the shell unitaries

USED BY IN THIS PROJECT

Faithfulness of the representation on the CAR trace Hilbert space

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Level 2

LEVEL 2

Faithfulness of the representation on the CAR trace Hilbert space

`MathlibAnnex.CStarAlgebra.CAR.separableCounterexampleRepresentation_injective`

Proves that the representation $\tilde{\rho} : T \rightarrow B(H_\tau)$ is injective.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The fixed algebra on the CAR trace Hilbert space

USED BY IN THIS PROJECT

A separably represented C*-algebra with no separable irreducible representation

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LEVEL 2

No nonzero irreducible representation on a separable Hilbert space

`MathlibAnnex.CStarAlgebra.CAR.not_isIrreducible_of_separable`

Rules out irreducible representations of T on separable Hilbert spaces by the finite-dimensionality theorem for a single irreducible representation class.

IMMEDIATE PREREQUISITES IN THIS PROJECT

The ambient representation of the fixed algebra, A simple C*-algebra with a unique irreducible representation class

USED BY IN THIS PROJECT

A separably represented C*-algebra with no separable irreducible representation

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Level 3

LEVEL 3

A separably represented C^* -algebra with no separable irreducible representation

MathlibAnnex.CStarAlgebra.CAR.atomicCounterexampleEndpoint_and_exists_separable_faithful_representation_and_no_separable_irreducible_representation

The same algebra T has a faithful representation on a separable Hilbert space, but no nonzero irreducible representation on any separable Hilbert space.

IMMEDIATE PREREQUISITES IN THIS PROJECT

No nonzero irreducible representation on a separable Hilbert space, Faithfulness of the representation on the CAR trace Hilbert space

USED BY IN THIS PROJECT

None in this scope

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