

Density and cardinality

The faithful separable representation gives the carrier bound $|T| \leq \mathfrak{c}$. Independently, the density obstruction gives $\mathfrak{c} \leq |D|$ for every norm-dense $D \subseteq T$. Taking $D = T$ proves that the norm density is exactly $\mathfrak{c} = 2^{\aleph_0}$ and supplies the continuum-density counterexample.

Carrier cardinality, norm-density character and Hilbert-space dimension remain separate notions. The existence statement allows a possibly nonunital algebra and does not include a faithful separable representation as one of its quantified requirements. Its chosen witness happens to be the fixed unital algebra T .

Exact Card references

- Exact norm density of the fixed atomic algebra —
MathlibAnnex.CStarAlgebra.CAR.hasDensityCharacter_atomicCounterexampleAlgebra
- A Naimark counterexample of continuum norm density —
MathlibAnnex.CStarAlgebra.CAR.existsNaimarkCounterexampleOfDensity_continuum

Boundary Inputs

The displayed edges preserve dependency paths through omitted helpers. Levels count selected predecessors within this scope. Mathematical citations remain distinct from formal dependencies.

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Dependency-first reading route

Levels belong to this reading scope. Follow prerequisites or uses to focus the route.

Level 0

LEVEL 0

Exact norm density of the fixed atomic algebra

MathlibAnnex.CStarAlgebra.CAR.hasDensityCharacter_atomicCounterexampleAlgebra

Combines a cardinality bound from a faithful representation on a separable Hilbert space with a lower bound for every norm-dense subset.

IMMEDIATE PREREQUISITES IN THIS PROJECT

None in this scope

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A Naimark counterexample of continuum norm density

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Level 1

LEVEL 1

A Naimark counterexample of continuum norm density

`MathlibAnnex.CStarAlgebra.CAR.existsNaimarkCounterexampleOfDensity_continuum`

Gives a C^* -algebra of norm density $\mathfrak{c}$ whose nonzero irreducible representations form one unitary-equivalence class, without an identification with the compact operators.

IMMEDIATE PREREQUISITES IN THIS PROJECT

Exact norm density of the fixed atomic algebra

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None in this scope

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