

# Rosenberg Compact-Operator Model

Let  $A$  be a nonzero complex  $C^*$ -algebra, not assumed unital, and let  $\pi$  act on a separable complex Hilbert space  $H$  as a representative of the unique unitary-equivalence class of nonzero irreducible representations of  $A$ . Then  $\pi$  is injective, every represented operator is compact, and every compact operator on  $H$  has an exact preimage in  $A$ .

Exact source: MathlibAnnex v0.4.0 · 63 declaration tiles · 11 levels

The result identifies  $A$ , through  $\pi$ , with the compact-operator model in an exact unbundled sense. The Project exposes the reusable proof route through faithfulness, character eigenvectors, countability, scalar corners, rank-one preimages and compact approximation.

$A$  is nonzero and may be genuinely nonunital.  $H$  is separable. The representation  $\pi$  is nonzero and irreducible, and every nonzero irreducible representation of  $A$  is unitarily equivalent to  $\pi$ . No separability of  $A$ , prior faithfulness or simplicity is assumed. The unital finite-dimensional consequences have their own exact hypotheses.

The 33 Card explanations have completed source-exposition correspondence review against exact MathlibAnnex v0.4.0 source. The compact-operator characterization is unbundled; no bundled StarAlgEquiv is supplied. The original 1953 paper and the Farah book remain unacquired and unreviewed in this record.

Thirty-three Declaration Card expositions are available. The added density route is source-only and creates no Card.

[Exact Project source](#)

[Exact source manifest](#)

## Proof architecture

**Faithfulness** — A pure state detecting a nonzero kernel element yields a nonzero irreducible GNS representation. Because every nonzero irreducible representation lies in the unique unitary-equivalence class represented by  $\pi$ , unitary equivalence forces the kernel element to act nontrivially, a contradiction.

**Characters and eigenvectors** — A character on a closed star subalgebra of the minimal unitization extends to a pure state. For a character distinct from the scalar character, the restricted GNS representation produces a unit joint eigenvector with that prescribed character.

**Countability and a scalar corner** — Distinct character eigenvectors are orthogonal. Separability of  $H$  makes the nonscalar character space countable; an isolated character then yields a nonzero projection whose corner is scalar.

**Every compact operator has a preimage** — A rank-one image and irreducible interpolation give exact preimages for operators of the form  $\text{rankOne}$ . Closed range and finite-rank approximation then give an exact preimage for every compact operator.

**Every represented operator is compact** — If some represented operator were noncompact, a character would separate a maximal abelian witness from the compact-preimage ideal. The resulting eigenvector conflicts with a rank-one preimage, so every image operator is compact.

**Unital finite-dimensional consequences** — For a unital algebra, the version quantifying competitors through the nonunital representation interface yields the same unique-class condition. Compactness of the identity image forces  $H$  to be finite-dimensional, then injectivity forces  $A$  to be finite-dimensional, giving the stated nonexistence consequence.

**Density below the continuum and the compact-operator model** — The compact-operator model route extends from separable representation spaces to an irreducible representation space with a norm-dense subset of cardinality strictly below the continuum. The nonunital statements retain the ordinary singleton-model hypotheses. Contrapositively, a singleton model that is not a compact-operator model forces every norm-dense subset of its representation space, and every norm-dense subset of the algebra, to have cardinality at least the continuum. This is a source-only extension of the reviewed 33-Card route; it creates no additional Cards. These statements are about the singleton irreducible model; the lower bound on its Hilbert space does not apply to arbitrary faithful reducible representations.

## Boundary Inputs

2690 exact Boundary Inputs: EXTERNAL\_COMPILED\_DECLARATION 2233, OMITTED\_INTERNAL\_NATIVE\_DECLARATION 457. Selected reachability is retained through every omitted internal declaration.

Eq — Compiled external provider used by selected Project declarations.

Membership.mem — Compiled external provider used by selected Project declarations.

PseudoMetricSpace.toUniformSpace — Compiled external provider used by selected Project declarations.

OfNat.ofNat — Compiled external provider used by selected Project declarations.

UniformSpace.toTopologicalSpace — Compiled external provider used by selected Project declarations.

And — Compiled external provider used by selected Project declarations.

Complex — Compiled external provider used by selected Project declarations.

Exists — Compiled external provider used by selected Project declarations.

Semiring.toNonAssocSemiring — Compiled external provider used by selected Project declarations.

Complex.instNormedField — Compiled external provider used by selected Project declarations.

Ring.toSemiring — Compiled external provider used by selected Project declarations.

Semiring.toMonoid — Compiled external provider used by selected Project declarations.

Set — Compiled external provider used by selected Project declarations.

AddCommMonoid.toAddMonoid — Compiled external provider used by selected Project declarations.

AddGroup.toSubNegMonoid — Compiled external provider used by selected Project declarations.

## Dependency-first reading route

### Level 0

#### **nonempty\_iInter\_of\_isCompact**

CantorScheme.nonempty\_iInter\_of\_isCompact

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: None in this Project

Used by in this Project: exists\_injective\_nat\_bool\_of\_isTopologicalBasis\_clopens

[Read exact source](#)

#### **Unbundled compact-operator model**

MathlibAnnex.Analysis.CStarAlgebra.IsCompactOperatorModel

Packages injectivity, compact-valuedness, and surjectivity onto all compact operators into one unbundled predicate.

Route / section: density · rosenberg

Immediate prerequisites in this Project: None in this Project

Used by in this Project: Compact-operator model from a representative of the unique irreducible-representation class, isCompactOperatorModel\_of\_singleton\_of\_dense, Compact-operator model from a unital representative of the unique irreducible-representation class

[Open Declaration Card](#)

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## Maximal abelian star subalgebra

MathlibAnnex.Analysis.CStarAlgebra.IsMaximalAbelian

Defines maximal abelian unital star subalgebras by commutativity and maximality under inclusion.

Route / section: countability-scalar-corner · density · every-compact · only-compact · rosenberg · unital-consequences

Immediate prerequisites in this Project: None in this Project

Used by in this Project: exists\_nonzero\_projection\_scalar\_corner\_of\_isolated\_character\_ne\_infinity, mem\_maximalAbelian\_of\_map\_eq\_rankOne\_of\_eigenvector, Finite-dimensional space for a unital representative of the unique irreducible-representation class, Maximal abelian subalgebra containing a self-adjoint element

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## Pure state as an extreme point

MathlibAnnex.Analysis.CStarAlgebra.IsPureState

Defines pure states as the real extreme points of the state space.

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · faithfulness · only-compact · rosenberg · unital-consequences

Immediate prerequisites in this Project: None in this Project

Used by in this Project: Pure-state extension of a character, Pure-state GNS irreducibility

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[Read exact source](#)

## Possibly nonunital representation

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation

Names nonunital complex star-algebra homomorphisms into bounded operators as the representation type.

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · faithfulness · only-compact · rosenberg

Immediate prerequisites in this Project: None in this Project

Used by in this Project: Nonunital irreducibility, Unitary equivalence of nonunital representations, character\_apply\_eq\_one\_of\_map\_eq\_rankOne, exists\_character\_ne\_infinity\_annihilating\_compact\_preimage, mem\_maximalAbelian\_of\_map\_eq\_rankOne\_of\_eigenvector

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## exists\_character\_ne\_infinity\_apply\_ne\_zero

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_character\_ne\_infinity\_apply\_ne\_zero

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: None in this Project

Used by in this Project: exists\_isolated\_character\_ne\_infinity\_of\_cardinalMk\_It\_continuum

[Read exact source](#)

## Scalar character of the minimal unitization

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.infinityCharacter

Defines the distinguished scalar-coordinate character of the minimal unitization.

Route / section: rosenberg

Immediate prerequisites in this Project: None in this Project

Used by in this Project: None in this Project

[Open Declaration Card](#)

[Read exact source](#)

## Unital representative of the unique irreducible-representation class

MathlibAnnex.Analysis.CStarAlgebra.Representation.IsSingletonIrreducibleModelAmongNonUnital

Defines the corresponding condition for a unital representation, with competing nonzero irreducible representations quantified through the nonunital representation interface.

Route / section: rosenberg · unital-consequences

Immediate prerequisites in this Project: None in this Project

Used by in this Project: Compact-operator model from a unital representative of the unique irreducible-representation class, Finite-dimensional algebra from a separable representative of the unique irreducible-representation class, Finite-dimensional space for a separable representative of the unique irreducible-representation class

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## inner\_eq\_zero\_of\_ne\_characters

MathlibAnnex.Analysis.CStarAlgebra.Representation.inner\_eq\_zero\_of\_ne\_characters

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: None in this Project

Used by in this Project: cardinalMk\_nonScalarCharacterSpace\_lt\_continuum\_of\_singleton\_of\_dense

[Read exact source](#)

## one\_le\_dist\_of\_norm\_eq\_one\_of\_inner\_eq\_zero

MathlibAnnex.Analysis.CStarAlgebra.Representation.one\_le\_dist\_of\_norm\_eq\_one\_of\_inner\_eq\_zero

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: None in this Project

Used by in this Project: cardinalMk\_nonScalarCharacterSpace\_lt\_continuum\_of\_singleton\_of\_dense

[Read exact source](#)

## Full operator image in finite dimension

MathlibAnnex.Analysis.CStarAlgebra.Representation.surjective\_of\_irreducible\_of\_finiteDimensional

Shows that an irreducible representation on a nonzero finite-dimensional Hilbert space has full operator image.

Route / section: rosenberg

Immediate prerequisites in this Project: None in this Project

Used by in this Project: Compact-operator model from a unital representative of the unique irreducible-representation class

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## A character separates a closed ideal

MathlibAnnex.Analysis.CStarAlgebra.exists\_character\_annihilating\_closedIdeal\_of\_not\_mem

Separates an element from a closed ideal of a commutative C\*-algebra by a character vanishing on the ideal.

Route / section: density · only-compact · rosenberg

Immediate prerequisites in this Project: None in this Project

Used by in this Project: exists\_character\_ne\_infinity\_annihilating\_compact\_preimage, Compact image of a representative of the unique irreducible-representation class

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## Algebraic compact-preimage ideal

MathlibAnnex.CStarAlgebra.compactPreimageIdeal

Collects the elements represented by compact operators into a two-sided algebraic ideal.

Route / section: density · only-compact · rosenberg · unital-consequences

Immediate prerequisites in this Project: None in this Project

Used by in this Project: exists\_character\_ne\_infinity\_annihilating\_compact\_preimage, Compact image of a representative of the unique irreducible-representation class, Finite-dimensional space for a unital representative of the unique irreducible-representation class

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## denseRange\_restrict\_of\_continuous

MathlibAnnex.Topology.denseRange\_restrict\_of\_continuous

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: None in this Project

Used by in this Project: exists\_dense\_cardinalMk\_lt\_continuum\_of\_continuous\_denseRange

[Read exact source](#)

### **exists\_injective\_into\_dense\_of\_separated**

MathlibAnnex.Topology.exists\_injective\_into\_dense\_of\_separated

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: None in this Project

Used by in this Project: cardinalMk\_le\_cardinalMk\_dense\_of\_separated

[Read exact source](#)

### **An isolated point in a countable Baire space**

MathlibAnnex.Topology.exists\_isOpen\_singleton

Extracts an isolated point from a nonempty countable T1 Baire space.

Route / section: countability-scalar-corner · every-compact · only-compact · rosenberg · unital-consequences

Immediate prerequisites in this Project: None in this Project

Used by in this Project: Isolated character away from infinity, Finite-dimensional space for a unital representative of the unique irreducible-representation class

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## **Level 1**

### **Nonunital irreducibility**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.IsIrreducible

Defines nonunital irreducibility as nonzero action with no nontrivial closed reducing subspaces.

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · faithfulness · only-compact · rosenberg

Immediate prerequisites in this Project: Possibly nonunital representation

Used by in this Project: Representative of the unique irreducible-representation class, denseRange\_apply\_of\_isIrreducible\_of\_apply\_ne\_zero

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### **Unitary equivalence of nonunital representations**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.UnitaryEquivalent

Defines unitary equivalence by an onto complex-linear isometry intertwining the two representations.

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · faithfulness · only-compact · rosenberg

Immediate prerequisites in this Project: Possibly nonunital representation

Used by in this Project: Representative of the unique irreducible-representation class

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### **character\_apply\_eq\_one\_of\_map\_eq\_rankOne**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.character\_apply\_eq\_one\_of\_map\_eq\_rankOne

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: Possibly nonunital representation

Used by in this Project: isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

[Read exact source](#)

### **exists\_character\_ne\_infinity\_annihilating\_compact\_preimage**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_character\_ne\_infinity\_annihilating\_compact\_preimage

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: Possibly nonunital representation, A character separates a closed ideal, Algebraic compact-preimage ideal

Used by in this Project: isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

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### **Isolated character away from infinity**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_isolated\_character\_ne\_infinity

Finds an isolated character distinct from the scalar character in a countable character space.

Route / section: countability-scalar-corner · every-compact · only-compact · rosenberg

Immediate prerequisites in this Project: An isolated point in a countable Baire space

Used by in this Project: Scalar corner from a representative of the unique irreducible-representation class

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### **exists\_nonzero\_projection\_scalar\_corner\_of\_isolated\_character\_ne\_infinity**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_nonzero\_projection\_scalar\_corner\_of\_isolated\_character\_ne\_infinity

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: Maximal abelian star subalgebra

Used by in this Project: exists\_nonzero\_projection\_scalar\_corner\_of\_singleton\_of\_dense

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### **mem\_maximalAbelian\_of\_map\_eq\_rankOne\_of\_eigenvector**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.mem\_maximalAbelian\_of\_map\_eq\_rankOne\_of\_eigenvector

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: Maximal abelian star subalgebra, Possibly nonunital representation

Used by in this Project: isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

[Read exact source](#)

## Maximal abelian subalgebra containing a self-adjoint element

MathlibAnnex.Analysis.CStarAlgebra.exists\_maximalAbelian\_containing\_isSelfAdjoint

Places a self-adjoint element inside a maximal abelian unital star subalgebra.

Route / section: countability-scalar-corner · density · every-compact · only-compact · rosenberg

Immediate prerequisites in this Project: Maximal abelian star subalgebra

Used by in this Project: Scalar corner from a representative of the unique irreducible-representation class, exists\_nonzero\_projection\_scalar\_corner\_of\_singleton\_of\_dense, isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

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## Pure-state extension of a character

MathlibAnnex.Analysis.CStarAlgebra.exists\_pureState\_extension

Extends a character of a closed unital star subalgebra to a pure state of the ambient algebra.

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · faithfulness · only-compact · rosenberg · unital-consequences

Immediate prerequisites in this Project: Pure state as an extreme point

Used by in this Project: Eigenvector for a character distinct from the scalar character, Pure state detecting a nonzero square

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## Pure-state GNS irreducibility

MathlibAnnex.Analysis.CStarAlgebra.isIrreducible\_pureState\_gnsStarAlgHom

Turns purity of a state into irreducibility of its canonical GNS representation.

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · faithfulness · only-compact · rosenberg · unital-consequences

Immediate prerequisites in this Project: Pure state as an extreme point

Used by in this Project: Eigenvector for a character distinct from the scalar character, Faithfulness of a representative of the unique irreducible-representation class, Simplicity from a faithful representative of the unique irreducible-representation class

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## cardinalMk\_le\_cardinalMk\_dense\_of\_separated

MathlibAnnex.Topology.cardinalMk\_le\_cardinalMk\_dense\_of\_separated

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_injective\_into\_dense\_of\_separated

Used by in this Project: cardinalMk\_lt\_continuum\_of\_separated\_of\_dense

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### **exists\_dense\_cardinalMk\_lt\_continuum\_of\_continuous\_denseRange**

MathlibAnnex.Topology.exists\_dense\_cardinalMk\_lt\_continuum\_of\_continuous\_denseRange

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: denseRange\_restrict\_of\_continuous

Used by in this Project: exists\_dense\_cardinalMk\_lt\_continuum\_of\_isIrreducible

[Read exact source](#)

### **exists\_injective\_nat\_bool\_of\_isTopologicalBasis\_clopens**

MathlibAnnex.Topology.exists\_injective\_nat\_bool\_of\_isTopologicalBasis\_clopens

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: nonempty\_iInter\_of\_isCompact

Used by in this Project: exists\_isOpen\_singleton\_of\_compactSpace\_of\_cardinalMk\_lt\_continuum

[Read exact source](#)

## **Level 2**

### **Representative of the unique irreducible-representation class**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.IsSingletonIrreducibleModel

Defines when a nonzero irreducible representation  $\pi$  is a representative of the unique unitary-equivalence class of nonzero irreducible representations of  $A$ .

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · faithfulness · only-compact · rosenberg

Immediate prerequisites in this Project: Nonunital irreducibility, Unitary equivalence of nonunital representations

Used by in this Project: Eigenvector for a character distinct from the scalar character, Faithfulness of a representative of the unique irreducible-representation class

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### **denseRange\_apply\_of\_isIrreducible\_of\_apply\_ne\_zero**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.denseRange\_apply\_of\_isIrreducible\_of\_apply\_ne\_zero

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: Nonunital irreducibility

Used by in this Project: exists\_denseRange\_apply\_of\_isIrreducible

[Read exact source](#)

## **Simplicity from a faithful representative of the unique irreducible-representation class**

MathlibAnnex.Analysis.CStarAlgebra.Representation.isSimpleCStarAlgebra\_of\_singleton\_of\_injective

Converts a faithful representative of the unique irreducible-representation class into closed-two-sided-ideal simplicity of the algebra.

Route / section: rosenberg · unital-consequences

Immediate prerequisites in this Project: Pure-state GNS irreducibility

Used by in this Project: Finite-dimensional space for a unital representative of the unique irreducible-representation class

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## **Pure state detecting a nonzero square**

MathlibAnnex.Analysis.CStarAlgebra.exists\_pureState\_nonzero\_on\_star\_mul\_self

Supplies a pure state that detects  $a^*a$ , enabling faithfulness arguments without separability of the algebra.

Route / section: density · every-compact · faithfulness · only-compact · rosenberg · unital-consequences

Immediate prerequisites in this Project: Pure-state extension of a character

Used by in this Project: Faithfulness of a representative of the unique irreducible-representation class, Finite-dimensional space for a unital representative of the unique irreducible-representation class

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[Read exact source](#)

## **cardinalMk\_It\_continuum\_of\_separated\_of\_dense**

MathlibAnnex.Topology.cardinalMk\_It\_continuum\_of\_separated\_of\_dense

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: cardinalMk\_le\_cardinalMk\_dense\_of\_separated

Used by in this Project: cardinalMk\_nonScalarCharacterSpace\_It\_continuum\_of\_singleton\_of\_dense

[Read exact source](#)

## **exists\_isOpen\_singleton\_of\_compactSpace\_of\_cardinalMk\_It\_continuum**

MathlibAnnex.Topology.exists\_isOpen\_singleton\_of\_compactSpace\_of\_cardinalMk\_It\_continuum

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_injective\_nat\_bool\_of\_isTopologicalBasis\_clopens

Used by in this Project: exists\_isOpen\_singleton\_of\_cardinalMk\_It\_continuum

[Read exact source](#)

## Level 3

### **exists\_denseRange\_apply\_of\_isIrreducible**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_denseRange\_apply\_of\_isIrreducible

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: denseRange\_apply\_of\_isIrreducible\_of\_apply\_ne\_zero

Used by in this Project: exists\_dense\_cardinalMk\_It\_continuum\_of\_isIrreducible

[Read exact source](#)

### **Eigenvector for a character distinct from the scalar character**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_unit\_eigenvector\_of\_character\_ne\_infinity

Produces a unit joint eigenvector whose eigenvalue character is the prescribed character distinct from the scalar character.

Route / section: characters-eigenvectors · countability-scalar-corner · density · every-compact · only-compact · rosenberg

Immediate prerequisites in this Project: Representative of the unique irreducible-representation class, Pure-state extension of a character, Pure-state GNS irreducibility

Used by in this Project: cardinalMk\_nonScalarCharacterSpace\_It\_continuum\_of\_singleton\_of\_dense, Countability of the full character space, isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

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### **Faithfulness of a representative of the unique irreducible-representation class**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.injective\_of\_singleton

Shows that a representative of the unique irreducible-representation class of a nonzero C\*-algebra is faithful.

Route / section: density · every-compact · faithfulness · only-compact · rosenberg

Immediate prerequisites in this Project: Representative of the unique irreducible-representation class, Pure state detecting a nonzero square, Pure-state GNS irreducibility

Used by in this Project: A represented rank-one projection from a representative of the unique irreducible-representation class, exists\_norm\_eq\_one\_and\_map\_eq\_rankOne\_of\_singleton\_of\_dense, isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

[Open Declaration Card](#)

[Read exact source](#)

## Finite-dimensional space for a unital representative of the unique irreducible-representation class

MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional\_space\_of\_singleton

Shows that the Hilbert space of a separably acting unital representative of the unique irreducible-representation class is finite-dimensional.

Route / section: rosenberg · unital-consequences

Immediate prerequisites in this Project: Maximal abelian star subalgebra, Simplicity from a faithful representative of the unique irreducible-representation class, Pure state detecting a nonzero square, Algebraic compact-preimage ideal, An isolated point in a countable Baire space

Used by in this Project: Compact-operator model from a unital representative of the unique irreducible-representation class, Finite-dimensional algebra from a separable representative of the unique irreducible-representation class, Finite-dimensional space for a separable representative of the unique irreducible-representation class

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## exists\_isOpen\_singleton\_of\_cardinalMk\_It\_continuum

MathlibAnnex.Topology.exists\_isOpen\_singleton\_of\_cardinalMk\_It\_continuum

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_isOpen\_singleton\_of\_compactSpace\_of\_cardinalMk\_It\_continuum

Used by in this Project: exists\_isolated\_character\_ne\_infinity\_of\_cardinalMk\_It\_continuum

[Read exact source](#)

## Level 4

### cardinalMk\_nonScalarCharacterSpace\_It\_continuum\_of\_singleton\_of\_dense

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.cardinalMk\_nonScalarCharacterSpace\_It\_continuum\_of\_singleton\_of\_dense

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: Eigenvector for a character distinct from the scalar character, inner\_eq\_zero\_of\_ne\_characters, one\_le\_dist\_of\_norm\_eq\_one\_of\_inner\_eq\_zero, cardinalMk\_It\_continuum\_of\_separated\_of\_dense

Used by in this Project: exists\_nonzero\_projection\_scalar\_corner\_of\_singleton\_of\_dense

[Read exact source](#)

### Countability of the full character space

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.countable\_characterSpace\_of\_nonUnital\_singleton

Shows that the full character space of a closed unitization subalgebra is countable when  $\pi$  is a separably acting representative of the unique irreducible-representation class.

Route / section: countability-scalar-corner · every-compact · only-compact · rosenberg

Immediate prerequisites in this Project: Eigenvector for a character distinct from the scalar character

Used by in this Project: Scalar corner from a representative of the unique irreducible-representation class

[Open Declaration Card](#)

[Read exact source](#)

### **exists\_dense\_cardinalMk\_It\_continuum\_of\_isIrreducible**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_dense\_cardinalMk\_It\_continuum\_of\_isIrreducible

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_denseRange\_apply\_of\_isIrreducible, exists\_dense\_cardinalMk\_It\_continuum\_of\_continuous\_denseRange

Used by in this Project: isCompactOperatorModel\_of\_singleton\_of\_dense\_algebra

[Read exact source](#)

### **exists\_isolated\_character\_ne\_infinity\_of\_cardinalMk\_It\_continuum**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_isolated\_character\_ne\_infinity\_of\_cardinalMk\_It\_continuum

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_character\_ne\_infinity\_apply\_ne\_zero, exists\_isOpen\_singleton\_of\_cardinalMk\_It\_continuum

Used by in this Project: exists\_nonzero\_projection\_scalar\_corner\_of\_singleton\_of\_dense

[Read exact source](#)

### **isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: character\_apply\_eq\_one\_of\_map\_eq\_rankOne, exists\_character\_ne\_infinity\_annihilating\_compact\_preimage, Eigenvector for a character distinct from the scalar character, Faithfulness of a representative of the unique irreducible-representation class, mem\_maximalAbelian\_of\_map\_eq\_rankOne\_of\_eigenvector, Maximal abelian subalgebra containing a self-adjoint element

Used by in this Project: isCompactOperator\_map\_of\_singleton\_of\_rankOne\_preimages

[Read exact source](#)

### **Compact-operator model from a unital representative of the unique irreducible-representation class**

MathlibAnnex.Analysis.CStarAlgebra.Representation.faithful\_and\_compactOperatorModel\_of\_singleton\_amongNonUnitals

Shows that a separably acting unital representative of the unique irreducible-representation class is faithful and has image exactly  $K(H)$ .

Route / section: rosenberg

Immediate prerequisites in this Project: Unbundled compact-operator model, Unital representative of the unique irreducible-representation class, Finite-dimensional space for a unital representative of the unique irreducible-representation class, Full operator image in finite dimension

Used by in this Project: None in this Project

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### Finite-dimensional algebra from a separable representative of the unique irreducible-representation class

MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional\_algebra\_of\_singleton\_amongNonUnital

Shows that a nonzero unital  $C^*$ -algebra is finite-dimensional when it has a representative of its unique irreducible-representation class on a separable Hilbert space.

Route / section: rosenberg · unital-consequences

Immediate prerequisites in this Project: Unital representative of the unique irreducible-representation class, Finite-dimensional space for a unital representative of the unique irreducible-representation class

Used by in this Project: No separably acting representative of all nonzero irreducible representations

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### Finite-dimensional space for a separable representative of the unique irreducible-representation class

MathlibAnnex.Analysis.CStarAlgebra.Representation.finiteDimensional\_space\_of\_singleton\_amongNonUnital

Shows that the Hilbert space of a separably acting representative of the unique irreducible-representation class is finite-dimensional.

Route / section: rosenberg

Immediate prerequisites in this Project: Unital representative of the unique irreducible-representation class, Finite-dimensional space for a unital representative of the unique irreducible-representation class

Used by in this Project: None in this Project

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## Level 5

### Scalar corner from a representative of the unique irreducible-representation class

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_nonzero\_projection\_scalar\_corner

Produces a nonzero projection with one-dimensional corner from a separably acting representative of the unique irreducible-representation class.

Route / section: countability-scalar-corner · every-compact · only-compact · rosenberg

Immediate prerequisites in this Project: Countability of the full character space, Isolated character away from infinity, Maximal abelian subalgebra containing a self-adjoint element

Used by in this Project: A represented rank-one projection from a representative of the unique irreducible-representation class

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### **exists\_nonzero\_projection\_scalar\_corner\_of\_singleton\_of\_dense**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_nonzero\_projection\_scalar\_corner\_of\_singleton\_of\_dense

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: cardinalMk\_nonScalarCharacterSpace\_It\_continuum\_of\_singleton\_of\_dense, exists\_isolated\_character\_ne\_infinity\_of\_cardinalMk\_It\_continuum, exists\_nonzero\_projection\_scalar\_corner\_of\_isolated\_character\_ne\_infinity, Maximal abelian subalgebra containing a self-adjoint element

Used by in this Project: exists\_norm\_eq\_one\_and\_map\_eq\_rankOne\_of\_singleton\_of\_dense

[Read exact source](#)

### **isCompactOperator\_map\_of\_singleton\_of\_rankOne\_preimages**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.isCompactOperator\_map\_of\_singleton\_of\_rankOne\_preimages

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: isCompactOperator\_map\_of\_isSelfAdjoint\_of\_singleton\_of\_rankOne\_preimages

Used by in this Project: isCompactOperatorModel\_of\_singleton\_of\_dense

[Read exact source](#)

### **No separably acting representative of all nonzero irreducible representations**

MathlibAnnex.Analysis.CStarAlgebra.Representation.not\_singleton\_amongNonUnital\_of\_infiniteDimensional

Shows that, for an infinite-dimensional unital C\*-algebra, no representation on a separable Hilbert space is a representative of the unique unitary-equivalence class of all nonzero irreducible representations.

Route / section: rosenberg

Immediate prerequisites in this Project: Finite-dimensional algebra from a separable representative of the unique irreducible-representation class

Used by in this Project: None in this Project

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[Read exact source](#)

## **Level 6**

### **A represented rank-one projection from a representative of the unique irreducible-representation class**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_nonzero\_projection\_rankOne\_map

Produces a nonzero projection whose represented image is a compact rank-one orthogonal projection.

Route / section: every-compact · only-compact · rosenberg

Immediate prerequisites in this Project: Scalar corner from a representative of the unique irreducible-representation class, Faithfulness of a representative of the unique irreducible-representation class

Used by in this Project: Preimages for all operators of the form rankOne

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### **exists\_norm\_eq\_one\_and\_map\_eq\_rankOne\_of\_singleton\_of\_dense**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_norm\_eq\_one\_and\_map\_eq\_rankOne\_of\_singleton\_of\_dense

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_nonzero\_projection\_scalar\_corner\_of\_singleton\_of\_dense, Faithfulness of a representative of the unique irreducible-representation class

Used by in this Project: exists\_preimage\_rankOne\_of\_singleton\_of\_dense

[Read exact source](#)

## **Level 7**

### **Preimages for all operators of the form rankOne**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_preimage\_rankOne\_of\_singleton

Provides preimages for every operator of the form  $\text{rankOne } \mathbb{C} \times y$ , the rank-at-most-one generators used to obtain all compact operators.

Route / section: every-compact · only-compact · rosenberg

Immediate prerequisites in this Project: A represented rank-one projection from a representative of the unique irreducible-representation class

Used by in this Project: Preimages of every compact operator, Compact image of a representative of the unique irreducible-representation class

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### **exists\_preimage\_rankOne\_of\_singleton\_of\_dense**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_preimage\_rankOne\_of\_singleton\_of\_dense

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_norm\_eq\_one\_and\_map\_eq\_rankOne\_of\_singleton\_of\_dense

Used by in this Project: isCompactOperatorModel\_of\_singleton\_of\_dense

[Read exact source](#)

## **Level 8**

### **Preimages of every compact operator**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.exists\_preimage\_of\_compact\_singleton

Establishes the inclusion  $K(H) \subseteq \pi(A)$  by giving a preimage for every compact operator.

Route / section: every-compact · rosenberg

Immediate prerequisites in this Project: Preimages for all operators of the form rankOne

Used by in this Project: Compact-operator model from a representative of the unique irreducible-representation class

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### **isCompactOperatorModel\_of\_singleton\_of\_dense**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.isCompactOperatorModel\_of\_singleton\_of\_dense

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: Unbundled compact-operator model, exists\_preimage\_rankOne\_of\_singleton\_of\_dense, isCompactOperator\_map\_of\_singleton\_of\_rankOne\_preimages

Used by in this Project: continuum\_le\_cardinalMk\_dense\_space\_of\_singleton\_of\_not\_isCompactOperatorModel, isCompactOperatorModel\_of\_singleton\_of\_dense\_algebra

[Read exact source](#)

### **Compact image of a representative of the unique irreducible-representation class**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.isCompactOperator\_map\_of\_singleton

Shows that every operator in the image of a separably acting representative of the unique irreducible-representation class is compact.

Route / section: only-compact · rosenberg

Immediate prerequisites in this Project: Preimages for all operators of the form rankOne, A character separates a closed ideal, Algebraic compact-preimage ideal

Used by in this Project: Compact-operator model from a representative of the unique irreducible-representation class

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## **Level 9**

### **continuum\_le\_cardinalMk\_dense\_space\_of\_singleton\_of\_not\_isCompactOperatorModel**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.continuum\_le\_cardinalMk\_dense\_space\_of\_singleton\_of\_not\_isCompactOperatorModel

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: isCompactOperatorModel\_of\_singleton\_of\_dense

Used by in this Project: None in this Project

[Read exact source](#)

### **Compact-operator model from a representative of the unique irreducible-representation class**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.faithful\_and\_compactOperatorModel\_of\_singleton

Shows that a representative of the unique irreducible-representation class on a separable Hilbert space is faithful and has image exactly  $K(H)$ .

Route / section: rosenberg

Immediate prerequisites in this Project: Unbundled compact-operator model, Preimages of every compact operator, Compact image of a representative of the unique irreducible-representation class

Used by in this Project: None in this Project

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### **isCompactOperatorModel\_of\_singleton\_of\_dense\_algebra**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.isCompactOperatorModel\_of\_singleton\_of\_dense\_algebra

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: exists\_dense\_cardinalMk\_lt\_continuum\_of\_isIrreducible, isCompactOperatorModel\_of\_singleton\_of\_dense

Used by in this Project: continuum\_le\_cardinalMk\_dense\_algebra\_of\_singleton\_of\_not\_isCompactOperatorModel

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## **Level 10**

### **continuum\_le\_cardinalMk\_dense\_algebra\_of\_singleton\_of\_not\_isCompactOperatorModel**

MathlibAnnex.Analysis.CStarAlgebra.NonUnitalCStarRepresentation.continuum\_le\_cardinalMk\_dense\_algebra\_of\_singleton\_of\_not\_isCompactOperatorModel

Exact source-only density bridge declaration; no Declaration Card is created.

Route / section: density

Immediate prerequisites in this Project: isCompactOperatorModel\_of\_singleton\_of\_dense\_algebra

Used by in this Project: None in this Project

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