Sphere Rigidity: Prior-Art Search

Sphere Rigidity ▪ R2 ▪ Search cutoff: 15 September 2026

Status of this version. This note records a bounded AI-assisted search. No equivalent result was identified in the sources examined through the cutoff date. This is not a novelty certificate or a guarantee that no earlier result exists.

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1. Claim boundary

The target statement concerns finite-dimensional real normed spaces. If the unit spheres of two such spaces are isometric for the ambient norm distance, then the spaces themselves are linearly isometric. This is an *object-level classification* statement. It does not assert that the prescribed sphere isometry extends linearly, and it does not claim a canonical extension.

That distinction matters. Tingley's marked extension problem asks whether a given surjective sphere isometry extends to a surjective real-linear isometry [1]. Kadets and Martín proved the marked extension result for finite-dimensional polyhedral spaces and explicitly separated the broader object-level question considered here [2, Section 5].

2. Problem lineage and closest inspected results

The problem-posing paper of Kadets and Martín is the primary historical anchor. It supplies both a substantial special case and an explicit formulation of the object-level question. Among the closest general results inspected in full text, Banakh proved the marked extension theorem for every two-dimensional real Banach space [3]. Cabello Sánchez proved a finite-dimensional local-to-global extension result under strict convexity and an additional local linearity hypothesis [4]. Other inspected results concern special algebraic or geometric classes, sequence spaces, Schatten classes, or invariants weaker than recovery of the complete linear-isometry type.

No inspected source was identified as stating or clearly implying the same theorem for all finite-dimensional real normed spaces.

3. Search record

The search began from Kadets–Martín, the references and selected citing literature around that paper, the bibliography of the current Sphere Rigidity manuscript, surveys and special-class results, recent preprints, and broader searches under adjacent formulations. The query vocabulary included Tingley's problem, sphere and boundary isometries, chord-metric reconstruction, convex-body rigidity, metric determination of normed spaces, approximate and phase isometries, and related formulations in p-normed spaces.

Search cutoff	15 September 2026
Recorded candidate sources	36
Candidate records classified	36 / 36
Same / clearly stronger / clearly implying	0 / 0 / 0
Unresolved candidate records	0
Owner-only access action required	None

All candidate records entered in the ledger were subsequently classified after theorem-level or full-text follow-up. No candidate remains unresolved in that finite ledger. The closest

additional papers concern marked extension under extra hypotheses, special Banach-space classes, geometric invariants, approximate whole-space isometries, or surveys. This does not mean that the mathematical literature has been exhausted; it means only that the candidates produced by the documented search no longer contain an unclassified possible overlap. No owner-only access request was required.

4. Method context

Several ingredients in the proof architecture have clear precedents. First-derivative minors as null Lagrangians and their boundary behavior occur in Ball–Currie–Olver [5]. Piola and determinant boundary identities are treated by Kupferman–Shachar [6]. Determinantal ideals and Plücker relations appear in Olver [7].

These are substantive method precedents, not prior proofs of the Sphere Rigidity theorem. No inspected source was identified that combines these ingredients into the same unit-sphere classification and norm-recovery route. This is a bounded method-history observation, not an originality certificate for every component.

5. Bounded finding and update rule

> **No equivalent result was identified in the sources examined through 15 September 2026.**

This sentence records the outcome of the documented search. It does not establish that no earlier result exists, and classification of the finite candidate ledger is not a proof of exhaustive coverage. If public release occurs substantially later than the cutoff, the search should receive a delta update or the cutoff should remain prominently displayed. Any newly identified close result should reopen the comparison regardless of the elapsed time.

The full technical audit retains the query log, candidate ledger, citation-chasing record, exact locators, classification rationales, and limitations. The public Project page should present only the bounded finding, the search cutoff, the problem-posing citation, the closest general inspected result, and a link to this note.

References

[1] D. Tingley, “Isometries of the unit sphere,” *Geometriae Dedicata* 22 (1987), 371–378. doi:10.1007/BF00147942.

[2] V. Kadets and M. Martín, “Extension of isometries between unit spheres of finite-dimensional polyhedral Banach spaces,” *Journal of Mathematical Analysis and Applications* 396 (2012), 441–447. doi:10.1016/j.jmaa.2012.06.031.

[3] T. Banakh, “Every 2-dimensional Banach space has the Mazur–Ulam property,” *Linear Algebra and its Applications* 632 (2022), 268–280. doi:10.1016/j.laa.2021.09.020.

[4] J. Cabello Sánchez, “A reflection on Tingley’s problem and some applications,” *Journal of Mathematical Analysis and Applications* 476 (2019), 319–336. doi:10.1016/j.jmaa.2019.03.041.

[5] J. M. Ball, J. C. Currie, and P. J. Olver, “Null Lagrangians, weak continuity, and variational problems of arbitrary order,” *Journal of Functional Analysis* 41 (1981), 135–174. doi:10.1016/0022-1236(81)90085-9.

[6] R. Kupferman and A. Shachar, “A geometric perspective on the Piola identity in Riemannian settings,” *Journal of Geometric Mechanics* 11 (2019), 59–76. doi:10.3934/jgm.2019004.

[7] P. J. Olver, “Hyperjacobians, determinantal ideals and weak solutions to variational problems,” *Proceedings of the Royal Society of Edinburgh Section A* 95 (1983), 317–340. doi:10.1017/S0308210500013020.